Mechanics 1

Version 1.0

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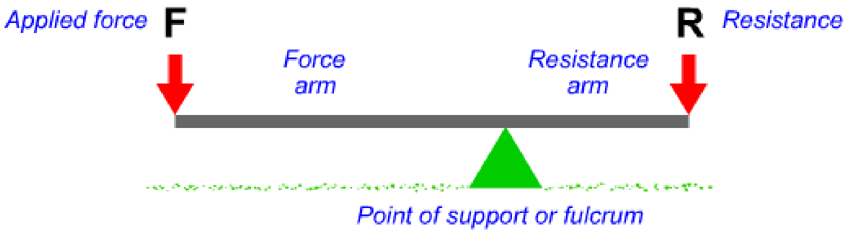
1. Levers

1.1. Levers are simple machines
Levers are simple machines. They are made up of a rigid bar and a point of support which is also called a fulcrum. They are normally used to exert a large force using only a small force, so they are a type of force multiplier. Using a sufficiently long imaginary lever, the force exerted by the weight of an ant could lift an elephant. However, one has to take into account the fact that this multiplication of force that the lever creates does not just happen by itself. In order to exert more force, the ant must move a long way, much further than the distance travelled by the elephant.

1.2. Mechanical advantage and mechanical disadvantage
When the lever multiplies the initial force, we can say that it has a mechanical advantage. Not all levers have a mechanical advantage. In some types of lever a large force has to be applied to overcome a small force and this is referred to as a mechanical disadvantage. The purpose of this type of lever is that it results in the small force moving a large distance, something which can be interesting in mechanisms such as barriers.
A barrier is a lever with mechanical disadvantage. One has to apply a lot of force at one end to overcome the small force in the other. In exchange, a large movement of the longer part is achieved.

1.3. Parts of a lever
The parts of a lever can be seen in this diagram:



Resistance is a force (many times the weight of the object) that has to be overcome by the use of another force, the applied force. The point of support, or fulcrum, is the point about which the lever swings. The arms, force arm and resistance arm, correspond to the distance between the fulcrum and the applied force or resistance.

1.4. The law of the lever
Levers behave according to a law of physics, called the Law of the Lever, which is expressed mathematically using the following equation. This is very useful in predicting how a particular lever will behave.

$$F \cdot B_F = R \cdot B_R$$

Where F is the applied force, B_F the length of the force arrm, R the resistance, and B_R the length of the resistance arm. Both the applied force and resistance should be expressed in newton (N). The length of the force arm and resistance arm are in meters (m).

1.5. Types of levers
Three types of levers exist according to the relative position of the force, the resistance and the point of support. They are:
- First class levers: The point of support, or fulcrum, is located between the force and the resistance. Example: Seesaw.
- Second class levers: The resistance is located between the force and the point of support. Example: Wheelbarrow.
- Third class levers: The force is located between the resistance and the point of support. Example: fishing rod.

4. Pulleys and hoists

4.1. Pulleys
Pulleys are wheels with the lateral surface adapted, usually in the form of a channel, so that a rope or a belt may come in contact with it without coming off. A single pulley does not multiply the applied force (it has no mechanical advantage), but it does change its direction, which is a big help, for example, to lift loads from the ground.

4.2. Hoists
A hoist is a combined set of pulleys that allows us to lift a great weight by applying a little strength. They are formed by fixed pulleys (that are anchored at the top) and movable pulleys (that are moving as the load stretches). The more pulleys a hoist has, less force must be exerted to lift the load (the higher the mechanical advantage), but also the greater the amount of rope that has to be pulled.
Every movable pulley of a hoist provides a mechanical advantage of 2, that is, halving the force that must be applied to overcome a particular resistance ($F = R / 2n$, where F is the applied force, R the resistance, and n the number of movable pulleys).

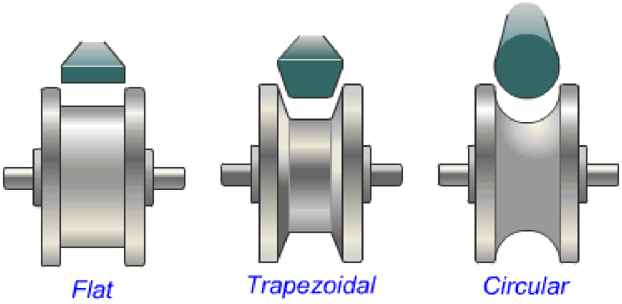
A person will be able to barely move a wheelbarrow of 60 kg with a single pulley. However, with a hoist of 3 movable pulleys, it will be necessary to apply a force which is 6 times lower (it will have to overcome an equivalent weight of only 10 Kg), so the wheelbarrow will be able to be lifted without problems.
The number of movable pulleys increases the mechanical advantage of a hoist. Different combinations of pulleys to lift a wheelbarrow of 60 Kg:
- With a single pulley I can barely move it; I'd better try with a hoist.
- With a hoist of a movable pulley, the equivalent weight of the wheelbarrow is 30 Kg. I can lift the wheelbarrow, but with a lot of effort.
- With a hoist of 2 movable pulleys, the equivalent weight of the wheelbarrow is 15 Kg. I can lift the wheelbarrow more easily, but I have to pull more rope.
- With a hoist of 3 movable pulleys, the equivalent weight of the wheelbarrow is 10 Kg. I can raise the wheelbarrow quite comfortably, although it is necessary to pull a lot of rope.

4.3. Windlass

A mechanism related to the pulleys is the windlass. It is a cylinder on which a cable or chain is wound. In its shaft there is a crank whose arm is longer than the diameter of the cylinder, allowing us to wind the cable into the cylinder using less force. The mechanical advantage of a windlass can be increased coupling a gear reduction mechanism.

4.4. Transmission through pulleys

If we join two pulleys through a belt, we can transmit the rotation movement of one pulley to the other. This is a transmission mechanism which is useful for machines that do not require much power. Is a transmission which is simple and economical to manufacture and maintain. There are different types of belts that can be used; on the right you can see the most common.



It is necessary that the pulleys are firmly joined to the shafts, to the motor shaft or to the shaft of the machine which is rotated. To ensure this union, normally a slit is made in the shaft and in the pulley and a piece of metal called cotter is introduced by pressure.

The pulley that transmits the movement is called motor or driving pulley and the pulley that receives the movement, output or driven pulley. There are three types of transmissions based on the diameter of both pulleys:
- Unitary transmission. If the two pulleys are equal, the output pulley will rotate at the same speed as the motor pulley.
- Multiplier transmission. If the output pulley is smaller than the motor one, it will rotate faster. It is a speed multiplication mechanism.
- Reductive transmission. If the output pulley is larger than the motor one it will rotate more slowly. It is a speed reduction mechanism.

The rotation speed of the output pulley can be calculated mathematically using the following equation:

$$Dm \cdot Nm = Ds \cdot Ns$$

Where Dm is the diameter of the motor pulley, Nm the speed of the motor pulley, Ds the diameter of the output pulley, and Ns the speed of the output pulley.
The "revolutions per minute" are used as a measurement unit, rpm for short.
It is possible to construct a gearbox placing several pairs of pulleys of different sizes in parallel. By changing the position of the belt, we can vary the speed of the output shaft. It is the mechanism that the table top drills usually have.

5. Gears

5.1. Introduction to gears

The gears or sprockets are mechanical elements designed to transmit turning movements. You can see them in many machines; its form is that of a wheel with teeth carved in its outline. These teeth engage - fit - in those of another similar wheel, in such a way that when one of them turns, it forces the other one to turn.
The fundamental characteristic of a gear is the number of teeth it has. This value is usually represented by the letter Z. So, if you read Z= 14, it means that the gear has 14 teeth. The other essential datum is the rotational speed, which is represented by the letter N. It is measured in rpm (revolutions per minute), and it indicates the number of turns a gear gives every minute.
In the gear mechanisms there is always one that pushes, the one called motor or driving gear, and another that receives the movement, the output or driven gear. The bar where the gears are mounted is the shaft.
By using gears we can easily change the rotational speed of a machine, for that we must use two gears with different number of teeth. If the output gear is larger than the motor gear, it will turn slower; if it is smaller, it will turn faster.

5.2. Types of gears

There are many types of gears. They can be classified into two groups according to the shape of the gear itself or according to the shape of the teeth.

According to the shape of the gear:

- Cylindrical gears: They have a cylinder shape. They transmit the movement between parallel shafts.
- Conical gears: They have a truncated cone shape. They transmit the movement between perpendicular shafts.

According to the shape of the teeth:

- Gears with straight teeth: The teeth follow straight lines (both the cylindrical and the conical).
- Gears with helical teeth: They have curved teeth. They are quieter than the gears with straight teeth, for this reason they are used in the gear boxes of cars. There are cylindrical helical gears and conical helical gears.

5.3. Functioning of the gears

Gears are used in the machines to transmit the rotation movement. We will study some of the aspects to consider when designing a mechanism with gears.

Two gears reverse the turning direction.

To transmit the movement, a couple of gears reverse the turning direction.

Idler gear

If you want to get the same rotational direction in the motor and in the output, you can insert an intermediate gear, called idler gear, which aims to reverse the turning direction.

Gear train

A mechanism formed by more than two gears is called a gear train.

Transformation of the movement: strength and speed

If a couple of gears has different sizes (different number of teeth), the rotational movement, as well as being transmitted, transforms. The speed and the strength that every gear will be able to transmit will be different. There are two possibilities: that it is a speed reducer or a speed multiplier mechanism.

- Speed reducer mechanism:

In this case the motor gear is smaller than the output gear. The output gear will turn more slowly, but it will be able to carry out more force. It's an interesting mechanism when we want to run a machine that must turn slowly with an engine that turns very fast, or when we have little force to do a job that needs a higher force. An example: a mechanism to manually upload a boat anchor.

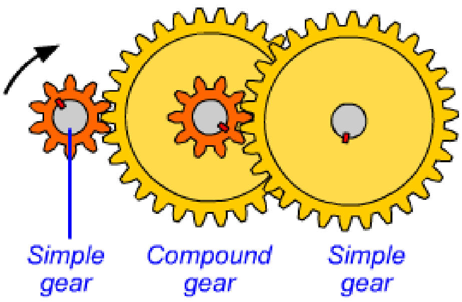
- Speed multiplier mechanism:

In this type of mechanism, the motor gear is bigger than the output gear. The output gear will rotate faster, but may carry out less force. It's an interesting mechanism when we want to run a machine very quickly. An example: a mechanism for manually operating an electric generator, as you can see in science labs or in some flashlights that work without batteries.

Compound gears

On numerous occasions the gears are used to reduce the speed of a motor. Sometimes, with a single pair of gears, it is impossible to reduce all the speed you need. Then, it is necessary to turn to the assembly of several pairs of consecutive gears. The best way to assemble these cascade gears is using compound gears: two gears of different sizes that are joined, moving, therefore, at the same speed.

In the image of the right a gear train with a compound gear is displayed. The driving gear (the one on the left) transmits the speed to a larger one (in yellow), that rotates slower, as it is larger. This is coupled with a small gear (the orange colored one in the center), forming a compound gear, which is coupled to a large output gear. The speed reduction in a mechanism as this is very large and it is achieved in a small space.



Gears transmit the movement accurately

There are other mechanisms that can be used with results similar to that of the gears, like the pulleys. Gears have the advantage that they transmit the movement accurately because they cannot slide, as sometimes happens with the pulleys, which may be very useful in some machines. However, in case of sudden engines starters the gear teeth can break, so using pulleys can be a good solution in simple machines.

Gears need lubrication

Gears need a lubricant (oil or grease) to function properly. It has three functions:

- To reduce friction, increasing the mechanical efficiency.
- To reduce the noise generated when the gears turn.
- To reduce the wear of the teeth, increasing the useful life of the gears.

6. Calculations in gears

6.1. Speed calculation in gears

In the gears, the transmission of movement is produced from tooth to tooth: when the motor gear (the one that pushes) advances one tooth, it forces the output one (the one pushed) to move another tooth. If both have the same number of teeth, they will rotate at the same speed. If the output gear has more teeth than the motor gear, it will rotate slower. If it has fewer teeth, it will rotate faster.

You can easily analyze the performance of a couple of gears, the data needed are:

Zm = number of teeth of the motor gear.

Nm = speed of the motor gear. It is usually measured in rpm, which means revolutions per minute.

Zs = number of teeth of the output gear.

Ns = speed of the output gear.

If you know 3 of these data, you can find the fourth using this formula: $Zm \cdot Nm = Zs \cdot Ns$

An example:

In a mechanism the motor gear has 10 teeth and rotates at 24 rpm. The output gear has 20 teeth. We do not know the speed of the output gear, but it's easy to find out. There are two ways: calculating or reasoning the operation.

a) Calculating:

1- We write the formula we need: $Zm \cdot Nm = Zs \cdot Ns$

2- We substitute the known values: $10\ teeth \cdot 24\ rpm = 20\ teeth \cdot Ns$

3- We clear: in this case Ns is the unknown to be cleared, the 'x' that is often used to resolve equations in mathematics: $Ns = \frac{10\ \cancel{teeth} \cdot 24\ rpm}{20\ \cancel{teeth}} = 12\ rpm$

b) Reasoning the operation:

The motor gear has 10 teeth. Every time it gives a full turn, it will push 10 teeth of the output gear, since the thrust is tooth by tooth. So, as the output gear has 20 teeth, it will only advance half a rotation: that is, for every rotation the motor gear gives, the output gear only gives half. Because the motor gear rotates at a speed of 24 rotations per minute (24 rpm), the output gear will only do half, that is, at 12 rpm.

Both by reasoning, and calculating, we have obtained the same solution. If this were not so we would have made a mistake we should locate.

6.2. Transmission ratio in gears

The transmission ratio (i) is a number that tells us how a mechanism transmits the rotation speed. If the transmission ratio is 2, the mechanism doubles the speed. If it is 1, it maintains the initial speed. If it is 0,5, it divides it in half. If it is 0,25, it divides it to a quarter. And so on.

The transmission ratio (i) can be calculated in two ways: by studying the sizes of the gears (the number of teeth) or through the study of their rotation speeds. In both cases the result must be the same. Consider an example: the calculation of the transmission ratio of the pair of gears, above.

a) Transmission ratio based on the size of the gears

The data required are:

Zm = number of teeth of the motor gear: 10 teeth.

Zs = number of teeth of the output gear: 20 teeth.

The formula that we must use is: $i = Zm / Zs$

We substitute the values and calculate: $i = 10\ \cancel{teeth} / 20\ \cancel{teeth} = 0{,}5$

The transmission ratio has no units: the ones from the numerator go with the ones of the denominator.

b) Transmission ratio from the rotation speed of the gears

The data required are:

Nm = speed of the motor gear: 24 rpm.

Ns = speed of the output gear: 12 rpm.

The formula that we must use in this case is: $i = Ns / Nm$

We substitute the values and calculate: $i = 12\ \cancel{rpm} / 24\ \cancel{rpm} = 0{,}5$

We must obtain the same solution as calculating from the number of teeth. If this were not so, we would have made a mistake.

The transmission ratio is very useful in complex mechanisms. Knowing the speed of the motor and the transmission ratio of the mechanism, we can easily find out what output speed we will have, without having to count the teeth on all of the gears composing the mechanism or make complex calculations. We must only use this formula: $Ns = Nm \cdot i$

8. Chain transmission

8.1. Chain transmission

In previous mini-units we have seen how the rotation movement is transmitted between gears. The chain transmission works in a similar way, with the advantage that the gears can be spaced from each other, which is very useful in many machines. The simplest transmission scheme by chain is formed by a motor sprocket (the one that pushes) and a driven or output sprocket (the one that receives the movement). The drag force is transmitted between the two wheels thanks to a chain, which is composed of many small articulated parts called links. The teeth of the sprockets used in this type of transmission have a shape designed to engage (fit) perfectly with the links of the chain.

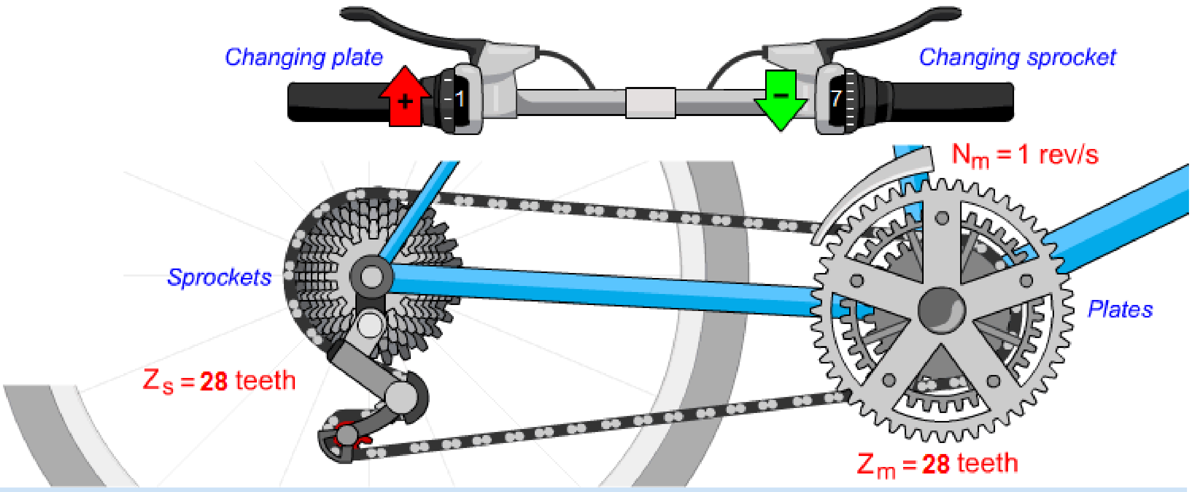
8.2. Movement equation in the chain transmission

The chain transmission follows the same movement equation as the gears, i.e.: $Zm \cdot Nm = Zs \cdot Ns$

8.3. Study of the gear of a bicycle

An example chain transmission is the one of the bicycle, which normally has a gear shift that allows adjusting the speed according to the inclination of the ground. Two types of sprockets act in the bicycle transmission: the plates and the idling sprockets. The plates are placed on the shaft of the pedals and spin when we pedal. The sprockets are located on the axis of the rear wheel and are smaller than the plates. The combination of the different plates and sprockets offers us a wide variety of suitable gears to climb, go on flat land or descend. The lower the speed of the bicycle, less will be the force we will have to apply to the pedals, they will be suitable gears for climbing. On flat land or downhill, we prefer gears that drive the bike quickly.

To calculate the speed of the bicycle we will need to know the perimeter of its rear wheel, the total length of its outline. The perimeter equation is: $P = 2 \cdot \Pi \cdot r$, where r is the wheel radius.



The rotation speed of the wheel (N_s) is:

$$N_s = \frac{Z_m \cdot N_m}{Z_s} = \frac{\mathbf{28}\ \text{teeth} \cdot 1\ \text{rev/s}}{\mathbf{28}\ \text{teeth}} = \mathbf{1\ rev/s\ (60\ rpm)}$$

The transmission ratio (i) is:

$$i = \frac{Z_m}{Z_s} = \frac{\mathbf{28}\ \text{teeth}}{\mathbf{28}\ \text{teeth}} = \mathbf{1}$$

The speed of the bicycle is obtained by multiplying N_s by the perimeter of the wheel (P):

$$V = N_s \cdot P = \mathbf{1\ rev/s} \cdot \mathbf{2.07\ m} = \mathbf{2.07\ m/s\ (7.45\ Km/h)}$$

