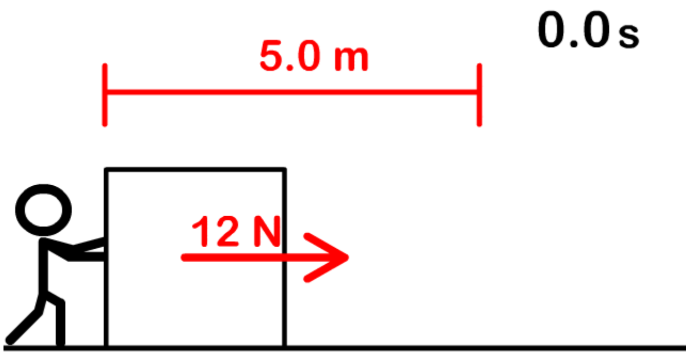
LESSON 5: WORK and ENERGY

Energy is a property that is related to changes or transformation processes in nature. Without energy no physical, chemical or biological process would be possible. The forms of energy associated with transformations of a mechanical type are called mechanical energy and their transfer from one body to another gets the name of 'Work'. Both concepts allow the study of the bodies' motion in a simpler way than using force terms and therefore constitute key elements in the description of physical systems.

1. Concept of Work in Physics.

In physics, work is the energy transferred to or from an object via the application of **force** along a **displacement**. In its simplest form, it is often represented as the product of force and displacement.

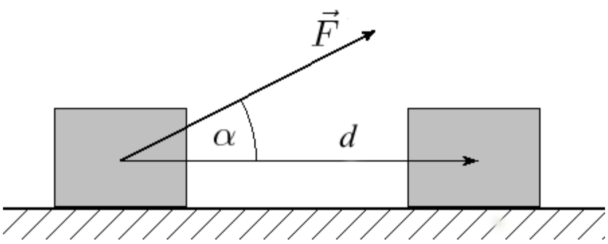


Source:https://stickmanphysics.com/stickman-physics-home/work-power-mechanical-energy-and-simple-machines/work-and-power/work-and-power-example-solutions/ [Requested on January the 15th of 2022]

$$W = F \cdot \Delta x = 12\ N \cdot 5\ m = 12\ J$$

whereas J stands for joules, the unit of work in the International System.

But what happens when the applied force is not parallel to the direction of motion? Then we have to reconsider this definition.



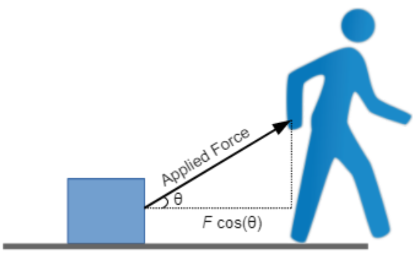
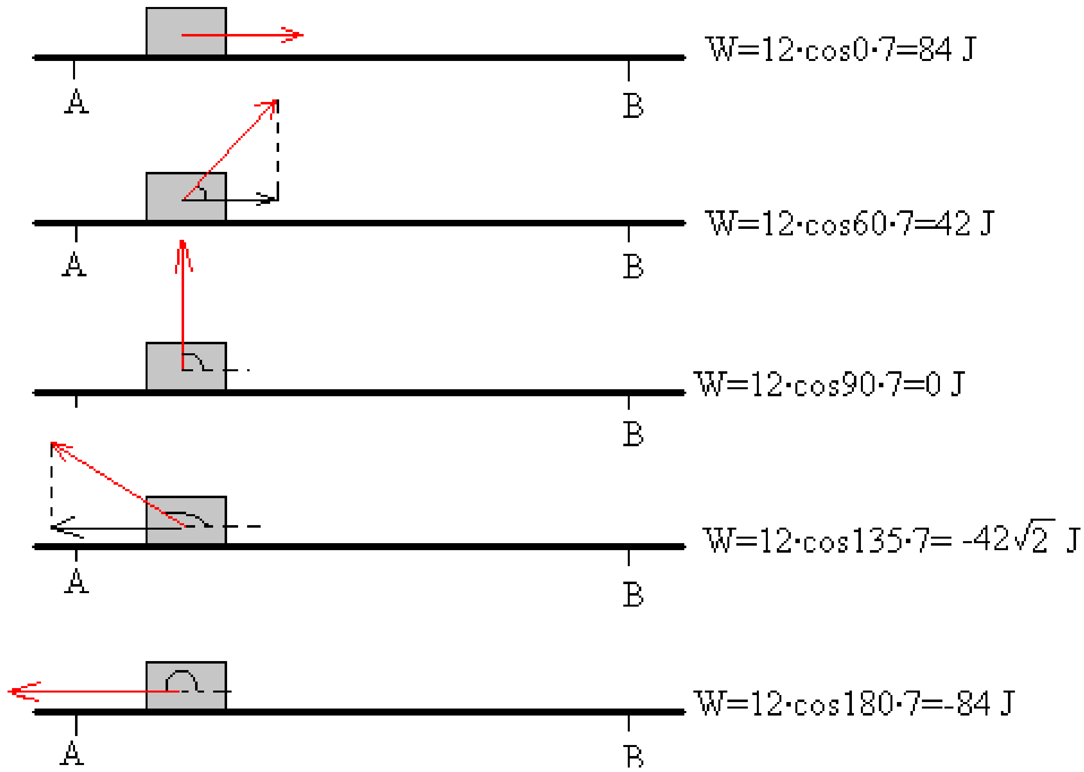


Image on the left: Author: Source:Ignacio Marcoux; Source: https://commons.wikimedia.org/wiki/File:Trabajo.png
Image on the right: Source: https://www.khanacademy.org/science/physics/work-and-energy/work-and-energy-tutorial/a/what-is-work
[Both requested on January the 15th of 2022]

$$W = F \cdot \Delta x \cdot \cos \alpha$$

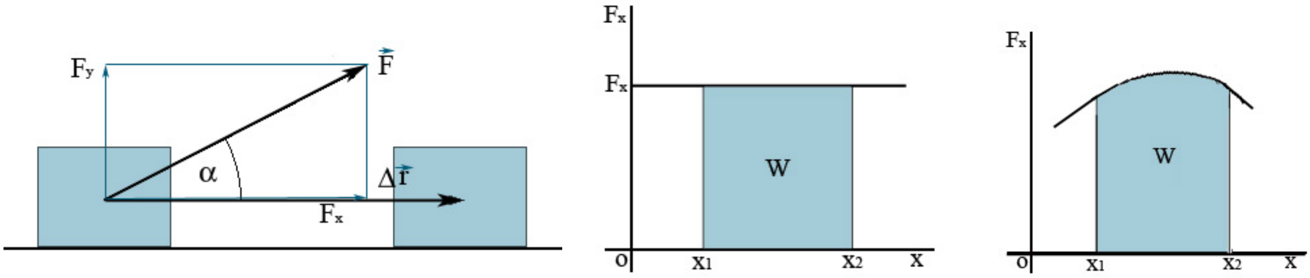
Example: calculate the work of a constant force of 12 N, whose point of application moves 7 m, if the angle between the directions of the force and the displacement are 0º, 60º, 90º, 135º, 180º.



Source: http://www.sc.ehu.es/sbweb/fisica/dinamica/trabajo/energia/energia.htm [Requested on January the 15th of 2022]

1.1. How to calculate the work done by a force from a chart?

- If the force is constant, $W = Fx \cdot \Delta x = Fx \cdot (x_2 - x_1)$. Check that the calculation matches that of area of a rectangle as drawn in the following graph:



Source: https://e-ducativa.catedu.es/44700165/aula/archivos/repositorio/4250/4340/html/11_trabajo_y_energa.html [Requested on January the 29th of 2022]

- But, **if the force is variable** the work can be calculated graphically from the closed area between the force and the start and end points.
    - We do not always know how to calculate, especially when the force varies in a nonlinear way (as in the case of from the graph above, although in the second year of high school you will know how to do it in this case as well).

2. Concept of Power in Physics.

The Power is a magnitude that give us the rate between the Work and the time. In other words, the Power is just the Work per unit of time.

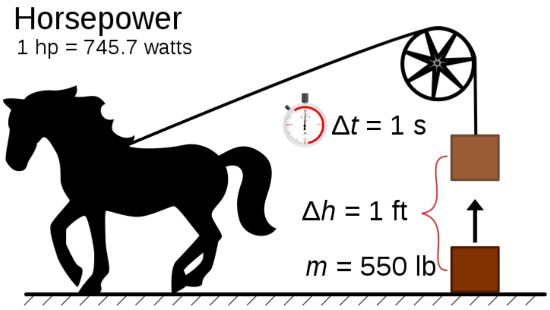
$$P = \frac{W}{t} = \frac{F \cdot \Delta x \cdot \cos\alpha}{t} = \frac{F_x \cdot \Delta x}{t} = F_x \cdot v_x$$

The units of Power in the International System are Watts [W]. So, 1 W = 1 J/s.

Long ago it was common to use steam horsepower.

Other units related to work and power are:

- The kilowatt/hour: 1 kWh = $3{,}6 \cdot 10^6$ J.



Author: User:Sgbeer; adapted by User:Martinvl; Source:https://en.wikipedia.org/wiki/File:Imperial_Horsepower.svg [Requested on January the 15th of 2022]

3. Concept of Energy in Physics.

Energy is a magnitude whose value depends on the position (potential energy) and/or the speed (kinetic energy) of a body. From these values it is possible to calculate the work done when an object is moving between two points, sometimes in an extremely simple way. There are different types of energy (kinetic, electrical, thermal, chemical, nuclear,...) but basically all the types of energy are reduced to two: kinetic and potential.

3.1. Kinetic energy. Theorem of the live forces.

Let's guess a body is moving on a horizontal surface and having a speed v_1. Then, the body gets the action of a force and, as a result of it, it acquires a speed v_2.

Because the force is horizontal, the angle between the force and the displacement is $\alpha = 0º$ and thus the work done by it can be calculated as:

$W = F \cdot \Delta x \cdot \cos\alpha = F \cdot \Delta x = m \cdot a \cdot \Delta x$ after applying the second Newton's law.

On the other hand, if the force we have is constant, so will be the acceleration as well. And that means that the body will follow a UARM and so we will be able to apply its equations.

From the third equation: $v_2^2 - v_1^2 = 2 \cdot a \cdot \Delta x$

We can isolate the acceleration multiplied by the displacement: $a \cdot \Delta x = \frac{v_2^2 - v_1^2}{2}$

And so: $W = m \cdot a \cdot \Delta x = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

If we now define the kinetic energy as $E_k = \frac{1}{2} m v^2$

We get the following very generic relationship, which is known as the theorema of the live forces:

$$W = \Delta E_k$$

which is always valid, regardless of the kind of forces that are acting on the body.

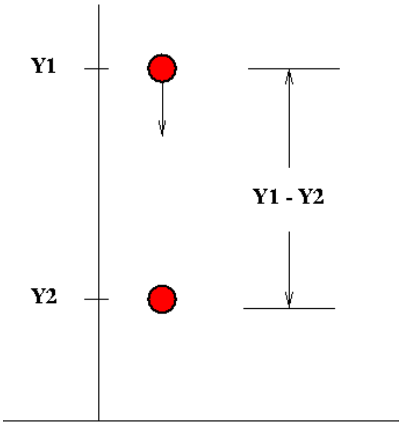
3.2. Potential energy.

The potential energy arises when we calculate the work done by some specific forces in such a way that it only depends on its value at the start and at the end pòint. The forces for which this can be done are called **conservative forces**.

3.2.1. Gravitational potential energy.

Gravitational force is one of these conservative forces. We will derive now the expression for the gravitational potential energy for a body close to the Earth's surface, from which the work done by the weight will be able to be calculated in a very simple way.

Imagine a ball is dropped from a height Y1 and we want to know the work done by the weight at the time it passes to the height Y2.



Source:https://theory.uwinnipeg.ca/mod_tech/node32.html [Requested on January the 15th of 2022]

For that purpose and taking into account that the weight is a constant force when in acts on objects near the surface of the Earth, P = m·g, we could proceed just as follows:

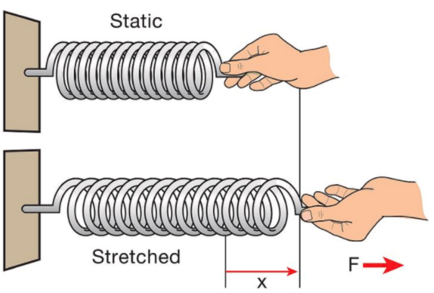
$$W = F \cdot \Delta y = -P \cdot (Y2 - Y1) = -(m \cdot g \cdot Y2 - m \cdot g \cdot Y1)$$

If we now define the gravitational potential energy as: $E_P^g = m \cdot g \cdot h$; whereas h can be Y1 or Y2 in this example. Then we can use it to express $W = -\Delta E_P^g$

3.2.2. Elastic potential energy.

The restoring elastic force is another of these conservative forces. We will now derive the expression of the restoring elastic force. This will be a bit more complex because, as Hooke's law, the restoring elastic force is not constant. It is for that reason that we will have to calculate the work done by this force from the area sustained by the force and the displacement, as we saw in subsection 1.1.

If we elongate a spring by applying an external force, the restoring elastic force will appear as a reaction and will be directed in the opposite direction.



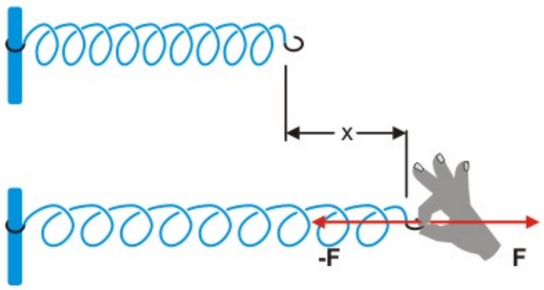
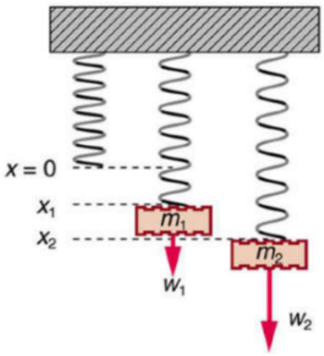


Image on the left Author: Science Photo Library; Source: https://pixels.com/featured/elastic-potential-energy-science-photo-library.html
Image on the left Author: David Hoult - The Open Door Team 2022; Source: https://www.saburchill.com/physics/chapters/0005a.html
[Both requested on January the 29th of 2022]

Now, if we apply an external force to a spring, initially elongated a distance x_1, and as a result of it the spring gets elongated until a distance x_2, the graph representation of the external force as a function of the elongation will look like this:



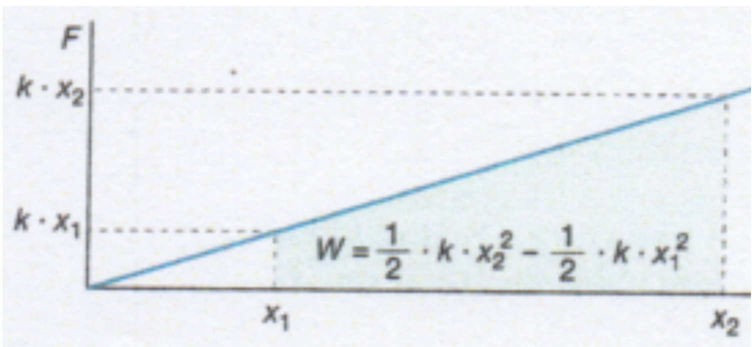


Image on the left Author: Raúl González Medina; Source: https://selectividad.intergranada.com/ESO/fq4/Clase/Tema_3_Trabajo.pdf [Requested on January the 29th of 2022]

So, the work associated with the external force in this process will be equal to the area sustained by the blue straight line and the horizontal axis between x_1 and x_2. And, as you can observa, this can be calculated as the area of the whole triangle from 0 to x_2 minus the area of the triangle from 0 to x_1.

Obviously the work associated with the restoring elastic force will be the same but just with a minus sign, because it acts in the opposite direction.

So, if we now define the elastic potential energy potential energy as: $E_P^e = \frac{1}{2}kx^2$; whereas x can be x_1 or x_2 in this example. Then we can use it to express $W = -\Delta E_P^e$

4. Principle of conservation of mechanical energy.

4.1. Without non-conservative forces (friction).

If we study a process in which the only forces acting are non-conservative ones, we can consider:

- on the one hand: $W = \Delta E_c$ (which is always valid, regardless of the force to be or not conservative)
- on the other hand: $W = -\Delta E_p$ (which is only valid when all the forces acting are conservatives)

Because both relations are referred to the same work, we just established:

$$\Delta E_c = -\Delta E_p$$
$$\Delta E_c + \Delta E_p = 0$$
$$\Delta(E_c + E_p) = 0$$
$$\Delta E_m = 0$$

whereas E_m stands for the mechanic energy and it is always equal to $E_m = E_c + E_p$.

We know that the kinetic energy is always calculated in the same way as pointed out in subsection 3.1.

But the potential energy can have multiple contributions, one for each force acting on the process which nature is the one of conservative forces (usually this apply for those forces pointing out to a center).

4.1.1. Example of a process with the only effect of the gravitational force.

Imagine you are on a roller coaster and you know that when your wagon passes at a height of 10 meters the speed it has is 2 m/s (see point A on image in the next page).

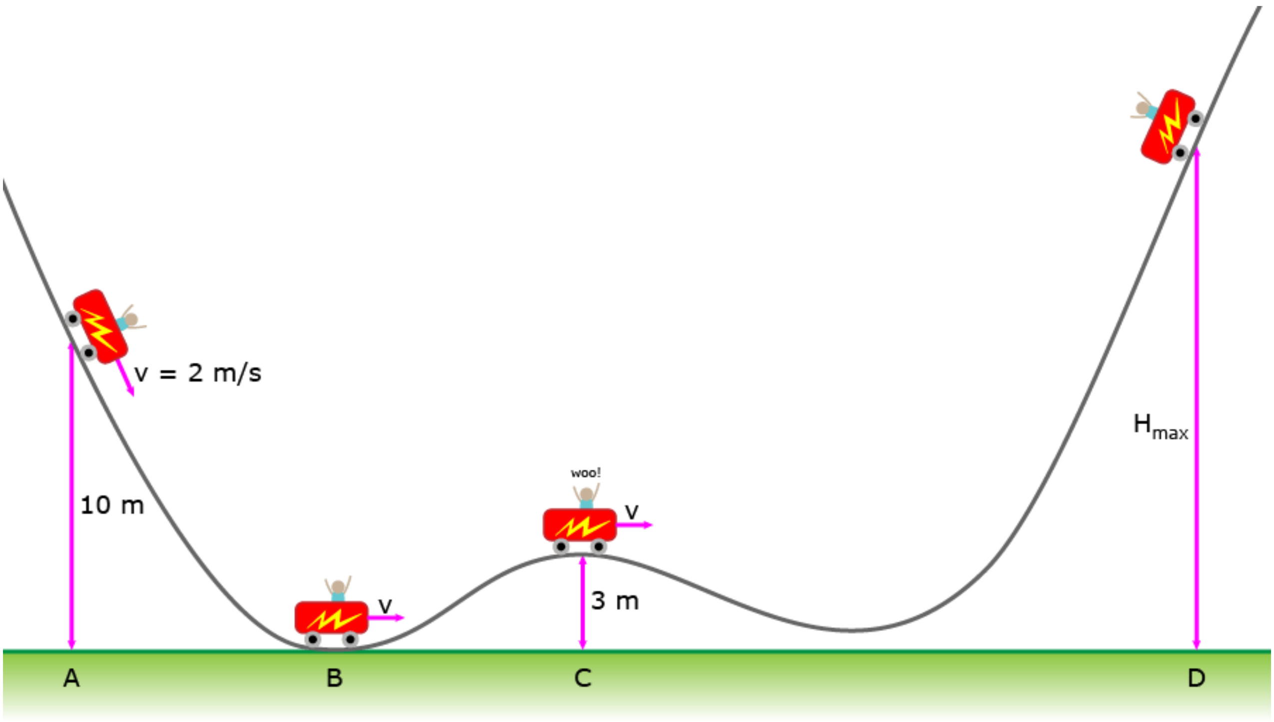
How could you determine the speed it has when it passes by the points B and C?

What would be the maximal height it could reach (H_{max} in C)?

Because the only force acting is the gravitational one and we know this is conservative, we can apply the principle of conservation of mechanical energy.

$$\Delta E_m = 0$$

which implies that $E_m^A = E_m^B = E_m^C = E_m^D$.



Author:Anne and Todd Helmenstine; Source:https://sciencenotes.org/potential-kinetic-energy-example-problem-work-energy-examples/ [Requested on January the 29th of 2022]

So, let's first calculate the mechanical energy in A:

$$E_m^A = E_c^A + E_p^A = \frac{1}{2}mv^2 + m\cdot g\cdot h = \frac{1}{2}\cdot m\cdot 2^2 + m\cdot 9,8\cdot 10 = 2\cdot m + 98\cdot m = 100\cdot m$$

Although we don't know the mass, we can use this expression to calculate the speed in B, since:

$$E_m^A = E_m^B$$

$$100\cdot m = E_c^B + E_p^B = \frac{1}{2}mv^2 + m\cdot g\cdot 0$$

$$100\cdot m = \frac{1}{2}mv^2$$

$$v = \sqrt{2\cdot 100} = 14,14\ m/s$$

In order to calculate the speed in C, we could proceed this way:

$$E_m^A = E_m^C$$

$$100\cdot m = E_c^C + E_p^C = \frac{1}{2}mv^2 + m\cdot 9,8\cdot 3$$

$$v = \sqrt{2\cdot(100 - 9,8\cdot 3)} = 11,88\ m/s$$

How could you proceed to calculate the maximal height wagon could reach at D? Thing about that and try to resolve it by yourself in your notebook.

4.1.2. Example of a process with the effect of both the gravitational and the restoring elastic force.

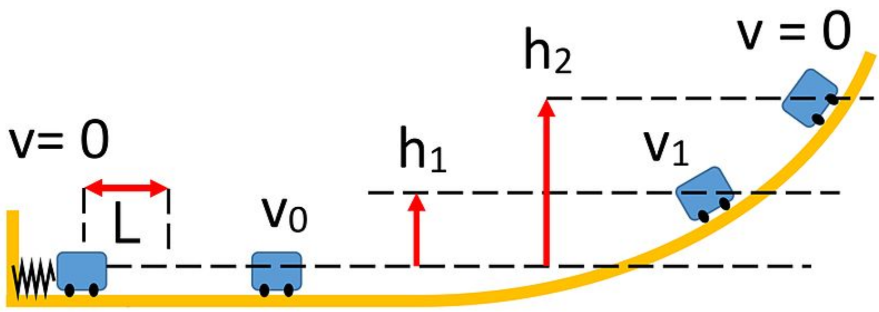
Imagine we drop a ball that falls vertically on a spring. What would be the value of the different kind of energies involved in the process at different times?

Author: José Luis Fernández y Gregorio Coronado; Source:https://www.fisicalab.com/apartado/energia-mecanica [Requested on January the 29th of 2022]

- Initially, the whole mechanical energy will be potential gravitational (see in green).
- As it falls, little by little the kinetic energy will increase (see in blue) as the potential gravitational energy decreases at the same rate. In other words, potential gravitational energy will be transferred to kinetic energy.
- Finally, one the ball reaches the spring, the potential elastic energy will start to increase (see in red), while both the kinetic and the potential gravitational energy will decrease.

But, as you can check, the total amount of mechanical energy remains constant all along the process,

Now, taking this into account, let's write one expression to find out the relation between the several parameters appearing in the following image:



Author:Guy vandegrift; Source:https://commons.wikimedia.org/wiki/File:Roller_coaster_energy_conservation.jpg [Requested on January the 29th of 2022]

$$m\cdot g\cdot h_2 = m\cdot g\cdot h_1 + \frac{1}{2}\cdot m\cdot v_1^2 = \frac{1}{2}\cdot m\cdot v_0^2 = \frac{1}{2}\cdot k\cdot L^2$$

4.2. With non-conservative forces.

When non-conservative forces act, the mechanical energy doesn't conserve anymore. But, in that case, we can affirm that the decrease of mechanical energy will be exactly equal to the work done by the non-conservative work.

$$\Delta E_m = W_{nc}$$

It is interesting to highlight that, in this course, the only non-conservative that we consider, is friction. But remember, the friction force in one that it is always acting in the opposite direction that the one along which the object is moving and because of that

$$W_{nc} = \mu \cdot N \cdot \Delta x \cdot cos\, 180^o = -\mu \cdot N \cdot \Delta x.$$

And this makes sense because we know that, due to friction, the total mechanical energy is going to decrease, because a part of it will be converted into heat.