LESSON 4: DYNAMICS II - UNIVERSAL GRAVITATIONAL AND HOOKE'S LAWS

1. The 3 Kepler's laws.

1.1. Astronomy evolution.

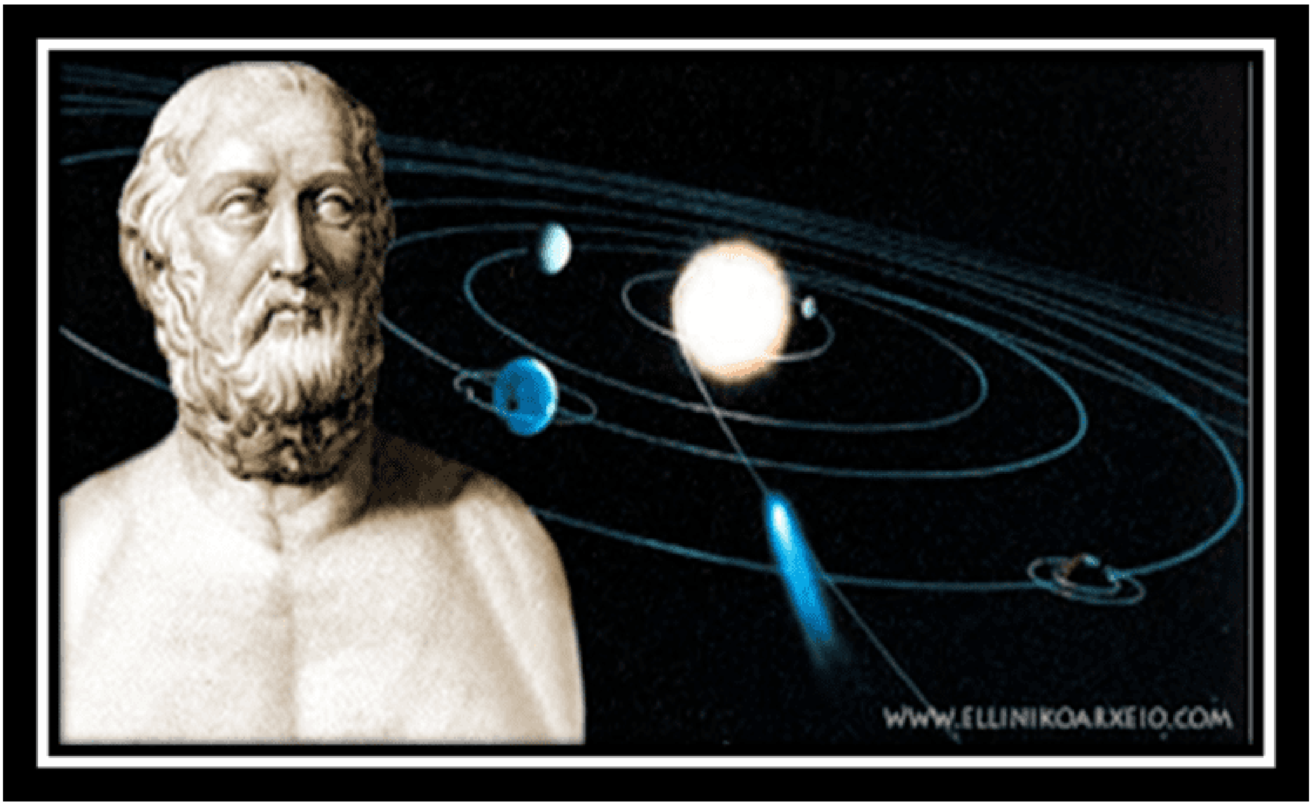
1.1.1. The ancient idea of the Universe.

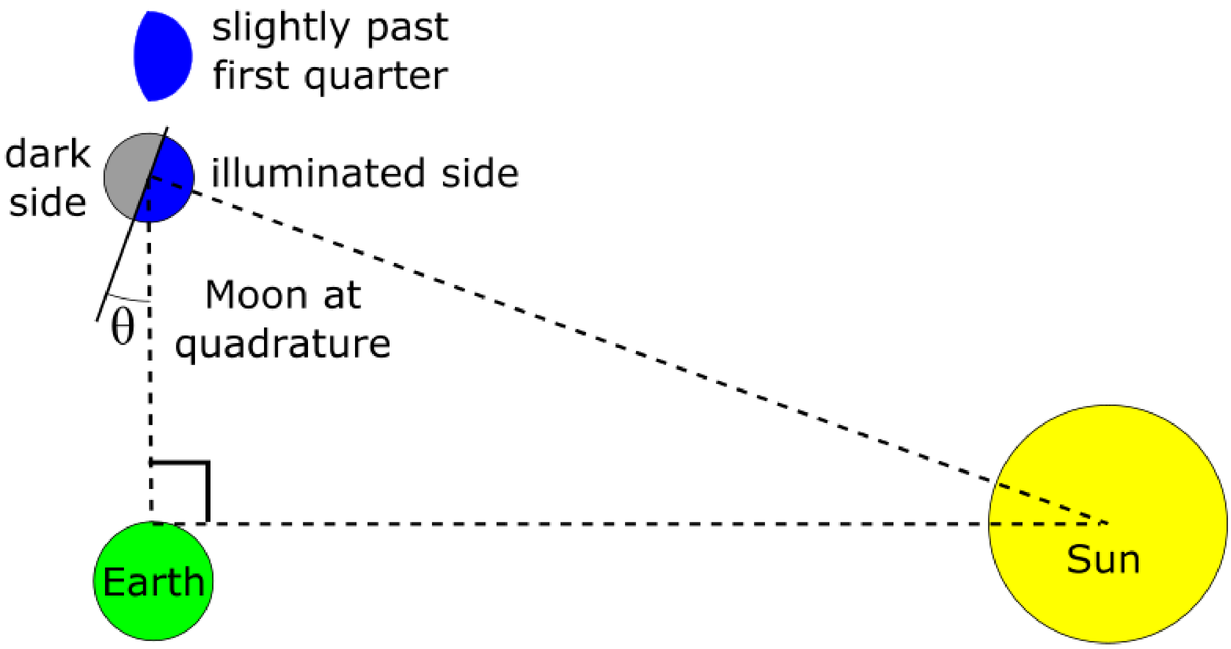
The ancient view of the Universe sustained by countries like ancient Greece or others even older was based on spheres inside of other spheres in a kind of multilayer model with the stars on the outside. The planets could be distinguished from the stars by their erratic motion.

Source:https://www.wikiwand.com/en/Ancient_Greek_astronomy [Requested on December the 27th of 2021]

1.1.2. Aristarchus of Samos: an old heliocentric model of the Universe.

But even in ancient Greece there was a man whose intuition pointed out the Sun, and not the Earth, to be at the center of the Universe: Aristarchus of Samos.

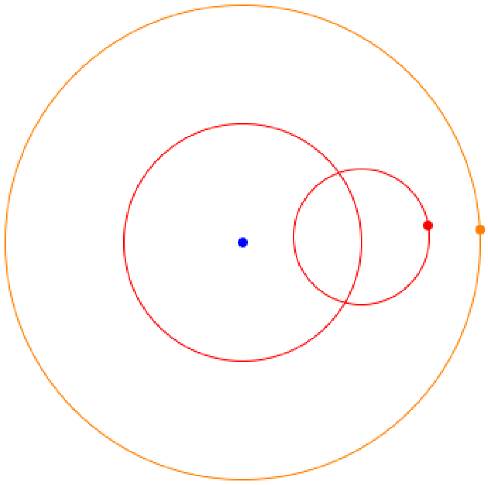




Author:Guy vandegrift, Source: https://commons.wikimedia.org/wiki/File:Aristarchus_distance_to_Sun_simple.svg [Requested on December the 27th of 2021]

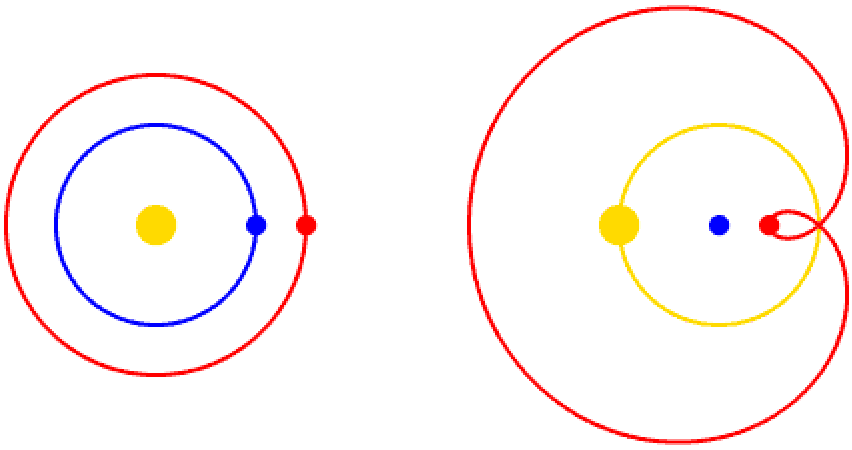
1.1.3. Heaven according to Ptolemy and the influence of Aristotle.

Although we actually know that the point of view of Aristarchus is basically correct, this was not the one we inherit from ancient Greece. And this is because later on, another important philosopher sustained a different point of view, based on the Earth being in the center of the Universe, and explaining the erratic motion of the planets[1] from a model including circles inside of circles.



Author:Ted Bunn, Source: https://blog.richmond.edu/physicsbunn/2012/09/13/ptolemy-and-copernicus/ [Requested on December the 28th of 2021]

This model was useful to explain apparent retrograde motion.



Author:Cleonis, Source: https://commons.wikimedia.org/wiki/File:Apparent_retrograde_motion.gif [Requested on December the 28th of 2021]

Other links to understand Ptolomy model based on epicycles and equants:

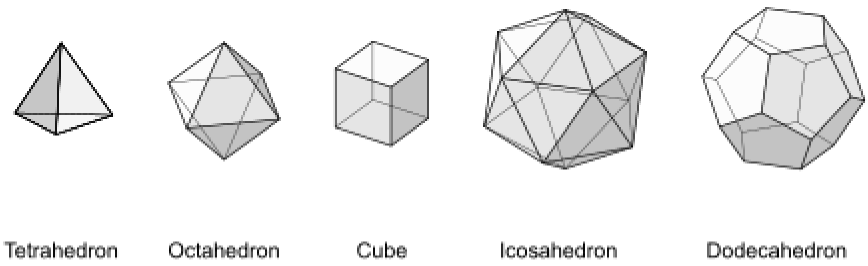
- http://ircamera.as.arizona.edu/NatSci102/NatSci/lectures/ptolemy.htm
- homework.uoregon.edu/pub/emj/121/lectures/ptolemy.html
- https://www.pinterest.es/pin/617908011351788436/

[1] In fact, planet means erratic in the ancient greek language.

1.1.4. Aristotelian prejudice and the 5 platonic solids.

We all know that Aristotle is a very famous philosopher from Ancient Greece. His popularity was so big that during centuries all the ideas defended by him were assumed as valid without discussion by most of the thinkers. But even though Aristotle introduced a lot of valid philosophic ideas, we actually know that not all of them were valid. Let's see a couple of examples:

- Aristotle assumed and defended as valid and defended the Ptolomy model of the Universe.
  - Some centuries later Copernic demonstrated that was wrong, resuming the old ideas from Aristarcus.
- He also thought that heavier objects would fall faster than lighter ones.
  - We know Galileo demonstrated that this is wrong, letting objects fall from Pisa's tower.
- He also sustained the still more ancient vision that the world, as to say, has to be divided in two worlds:
  - the underworld, where we live, based on unperfect laws.
  - the celestial world: where divinities live, based on perfect laws. These perfect laws should be based on perfect geometric figures, among them the circle and the 5 platonic solids.

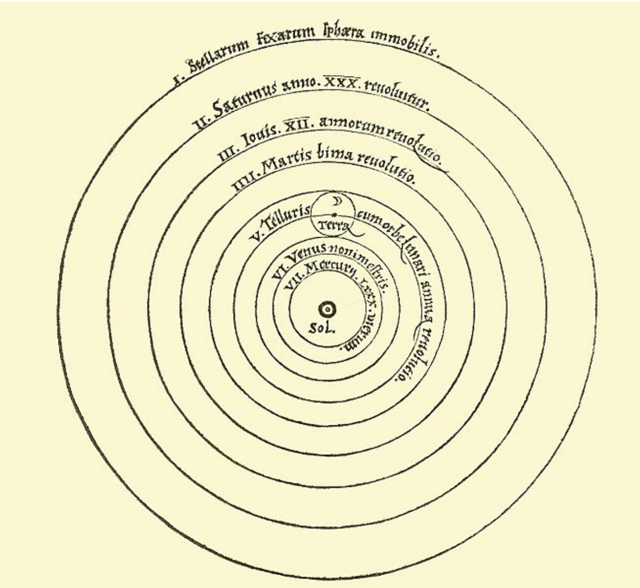


Author:David Eric Ffell , Source:https://commons.wikimedia.org/wiki/File:Platonic_Solids_Composition_51.svg [Requested on December the 28th of 2021]

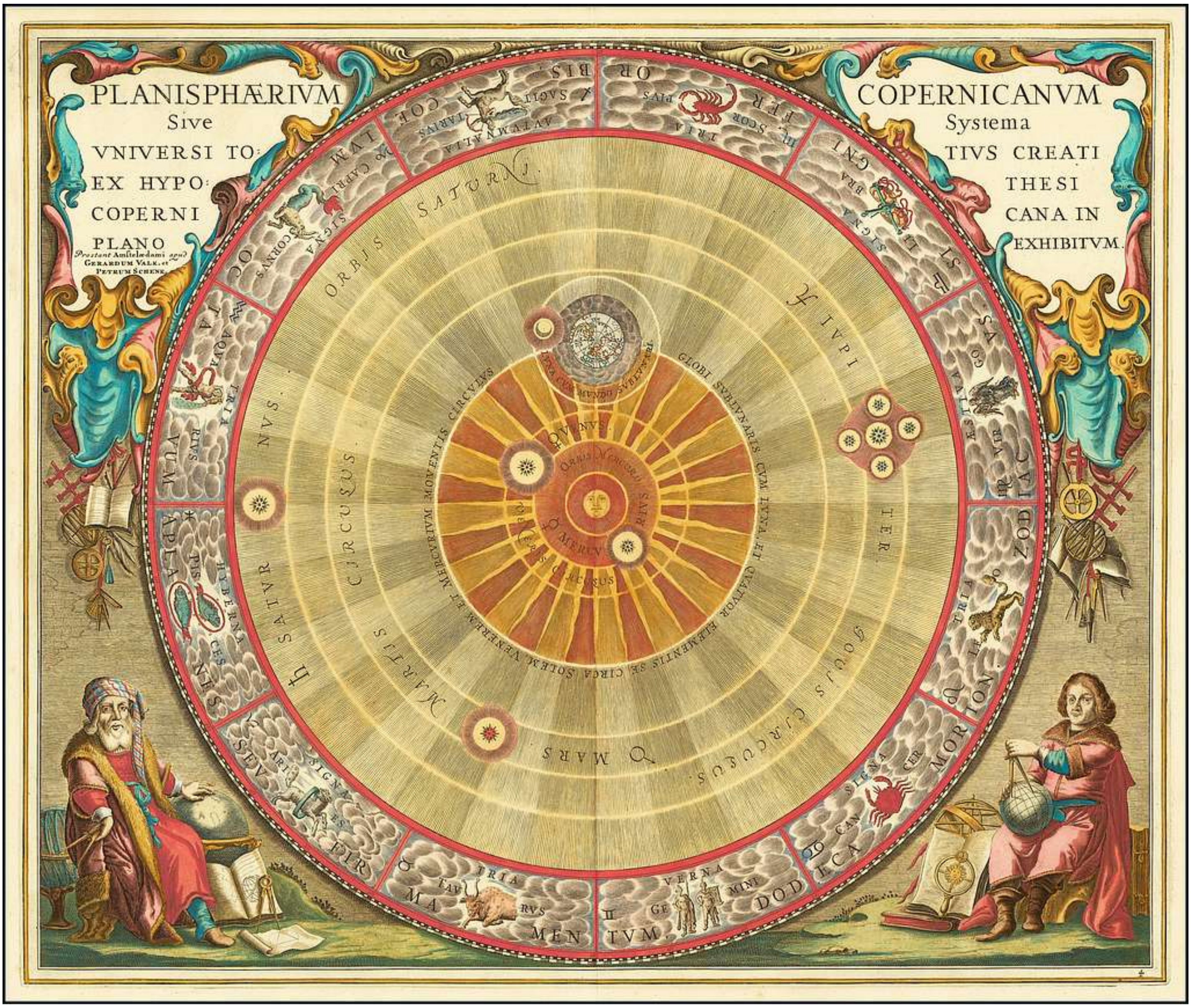
1.1.5. Copernican twist.

In 1543 Copernicus published a "new" astronomical model replacing the Sun in the Center of the Universe.

Although he had circulated an outline of his own heliocentric theory to colleagues sometime before 1514, he did not decide to publish it until he was urged to do so later by his pupil Rheticus.



Author Professor marginalia; Source:https://en.wikipedia.org/wiki/File:Copernican_heliocentrism_diagram-2.jpg [Requested on December the 29th of 2021]



Source: https://jenikirbyhistory.getarchive.net/amp/media/1660-copernican-astronomical-chart-in-the-form-of-the-concentric-circles-ee8068
[Public domain image requested on December the 29th of 2021]

1.2. Galileo Galilei and the first telescope.

The invention of the telescope is attributed to Galileo although, the only thing he did, was to improve a model of spyglass arrived to Venice, carried by ducht sellers. In fact, the patent of the spyglass is attributed to the dutch inventor Hans Lippershey. Anyway, recent investigations point out the catalan opticien from Girona, Joan Roget, to be the inventor of the primer prototype of skyglass, although he didn't proceed to register the patent.

Author Terapeak; Source:https://www.pinterest.es/pin/312226186659858671/ [Requested on December the 29th of 2021]

If the contribution of Galileo was so important, was because, on the one hand, he improve a lot the potence (lateral raise) of the prototype he got, and, on the other hand, he had the idea to use it to point out the sky.

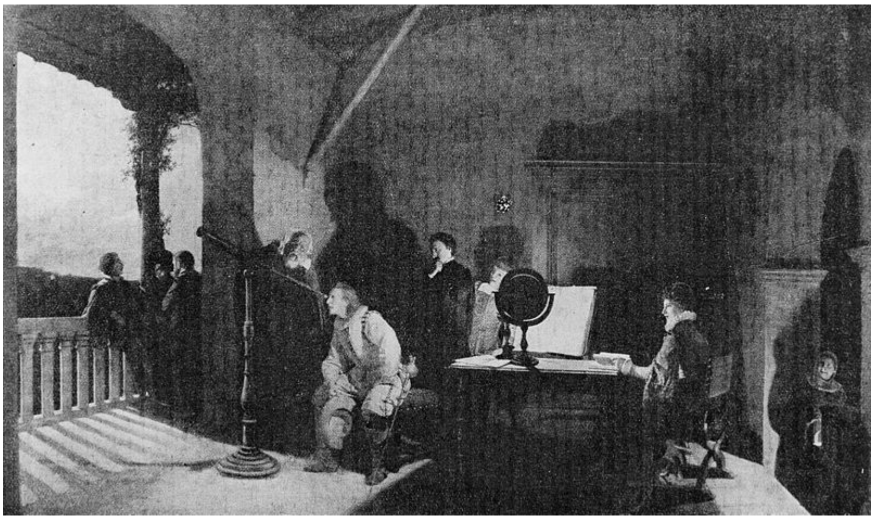
 

Image on the left: Author: Imatge de domini públic de David J Wilson;
Source: https://commons.wikimedia.org/wiki/File:Bertini_fresco_of_Galileo_Galilei_and_Doge_of_Venice.jpg
Image on the right: Author: Fæ;
Source: https://commons.wikimedia.org/wiki/File:Galileo_showing_to_John_Milton_the_markings_on_the_moon._Wellcome_M0004626.jpg
[Both requested on December the 29th of 2021]

Thanks to that, **Galileo reinforced the Copernician heliocentric point of view**, due to all the phenomena that he could observe, among which we highlight the following ones:

- the mountains of the moon (oh!...the sky is not perfect!!).
- a bunch of new stars.
- the satellites of Jupiter.
- the phases of Venus.
- the argument of the tides.
- Sun spots.

1.3. Tycho Brahe and the major estelar catalogue of all time.

Tycho Brahe was a Danish nobleman, fond of mathematics and astronomy (in addition to excesses such as banquets and parties), to whom the King of Sweden gave him an island with a palace that contained the largest telescope of the time.

With this telescope Brahe made astronomical measurements for many years with which he made the largest astronomical catalog of the time.

His plans were to give the most valuable data in his catalog to his son to analyze for important findings. But his son had the same hobbies as his father and was unable to focus enough on mathematics to complete an accurate analysis of those data.

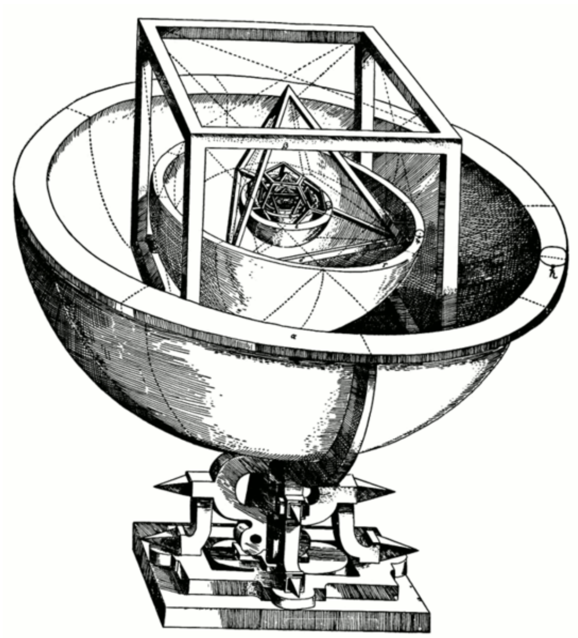
So, already on his deathbed, it is not clear whether out of repentance, he finally gave those data to his tenacious and brilliant disciple Johannes Kepler, for further study. Or if Kepler, taking advantage of Brahe's weakness, stole them.

Image on the left: Author: Science Photo Library; Source: https://www.bbc.com/mundo/noticias-40964869
Image on the right: Author: Andrzej Otrębski; Source: https://commons.wikimedia.org/wiki/File:Praha_Kepler_Brahe_2.jpg
[Both requested on January the 4th of 2022]

1.4. Johannes Kepler and the study of Mars's orbit around the Sun.

1.4.1. Kepler's Platonic Prejudice.

The deep conviction about the validity of the universe vision proposed by Aristotle, based on the 5 Platonic solids, and in which the laws of the sky had to be perfect, led Kepler to try fitting the orbits of the planets around the Sun, in a model that today is not plausible. In particular, his greater effort was focused in the analysis of Mars orbit trying to fit it in a circle within one of the platonic solids. But finally he had to admit that this fitting didn't match.



Author Johannes Kepler;
Source: https://commons.wikimedia.org/wiki/File:Kepler-solar-system-1.png
[Public domain image requested on January the 4th of 2022]

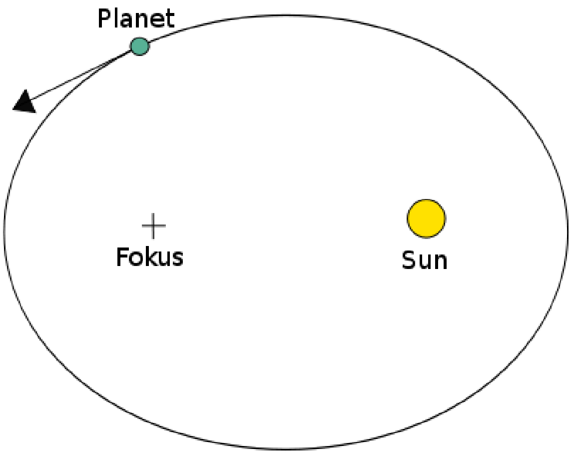
1.4.2. The laws of Kepler.

Johannes Kepler had to do a huge mathematical effort based on trigonometry to manually calculate the orbit of Mars around the Sun, which led him to find out that these were based on **ellipses**[2] and not circles, despite being much more inclined initially for the later.

1.4.2.1. The first Kepler's law.

Each planet's orbit about the Sun is an ellipse. The **Sun's center** is always **located at one focus** of the orbital ellipse. The planet follows the ellipse in its orbit, meaning that the planet to Sun distance is constantly changing as the planet goes around its orbit.

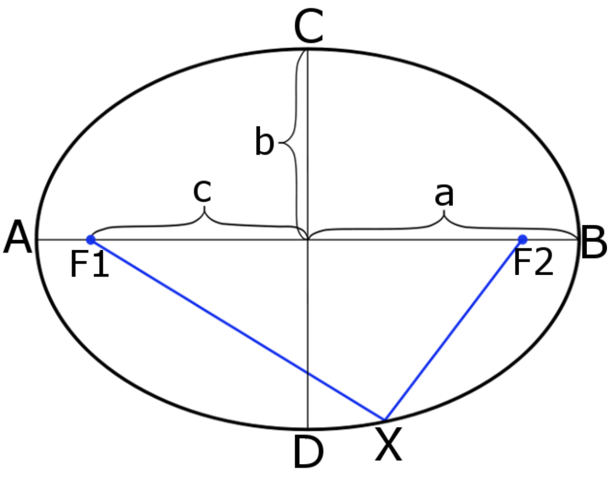
So, what the hell is an ellipse?



AuthorTalifero; Source:https://commons.wikimedia.org/wiki/File:Kepler%27s_law_1_en.svg [Public domain image requested on January the 4th of 2022]

- How to draw an ellipse?
- The conic sections.
- Conic Section 3D Animation.
- Real Conic Sections (Ellipse, Circle, Parabola, Hyperbola)
- **Definition** and **parameters** of an ellipse.

  An ellipse is defined as a geometric place that contains all those points whose sum of the distances to two fixed points called focus (F1 and F2) is constant.



Author ZetaZeti; Source:https://it.wikipedia.org/wiki/File:Ellisse.png [Public domain image requested on January the 4th of 2022]

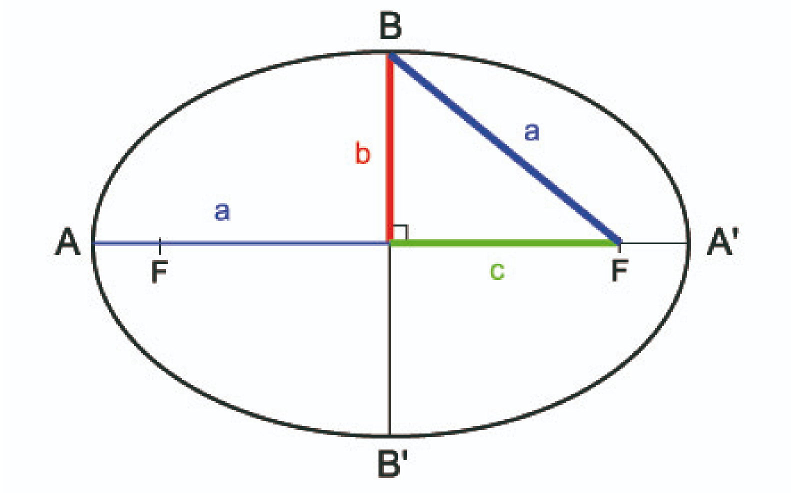
The ellipse is characterized by three main parameters:

- ☐ **a: the major semi axis**. Distance from the farthest point to the center of the ellipse.
- ☐ **b: the minor semi axis**. Distance from the point closest to the center of the ellipse.
- ☐ **c: focal length**. Distance from each focus to the center of the ellipse.

[2] We are going to see in the next subsection what an ellipse is.

There are two main relationships that can be found between the parameters of an ellipse:

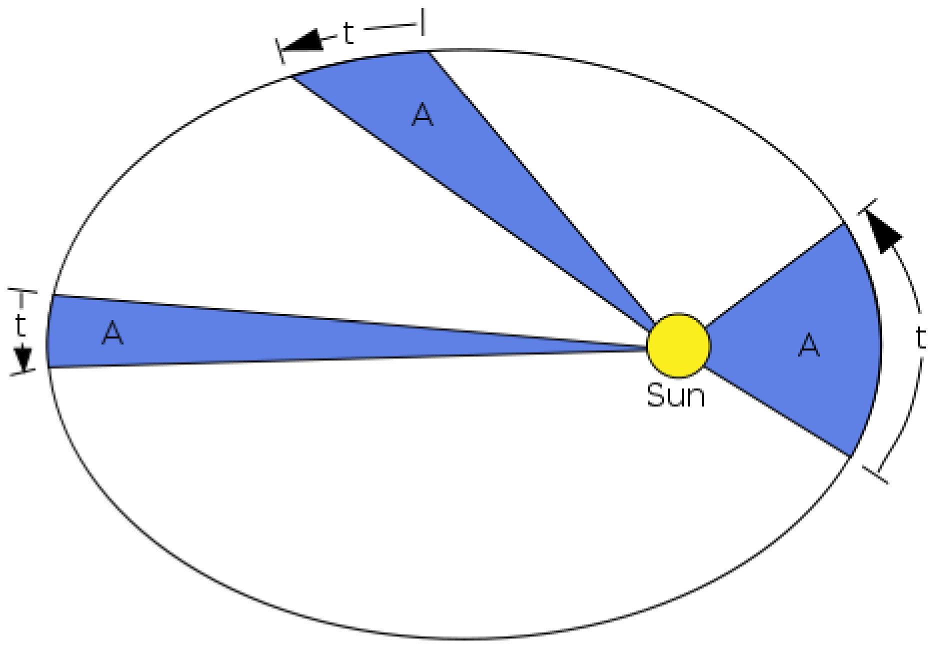
1. This constant value equal to the sum of the distances of any of its points to F1 and F2, in contraction, must be equal to 2a. This can be seen more easily if you think about what this sum is worth if the chosen points are A or A'.
2. The relationship between a, b and c is that of a right triangle, as shown in the following image. This can be seen more easily if you think about the value of the distance between B to either focus (F or F'), given that these two points are equidistant.



Author: Moran-Tao; Source:https://es.wikipedia.org/wiki/Archivo:Elipse1.0.jpg [Requested on January the 4th of 2022]

1.4.2.2. The second Kepler's law.

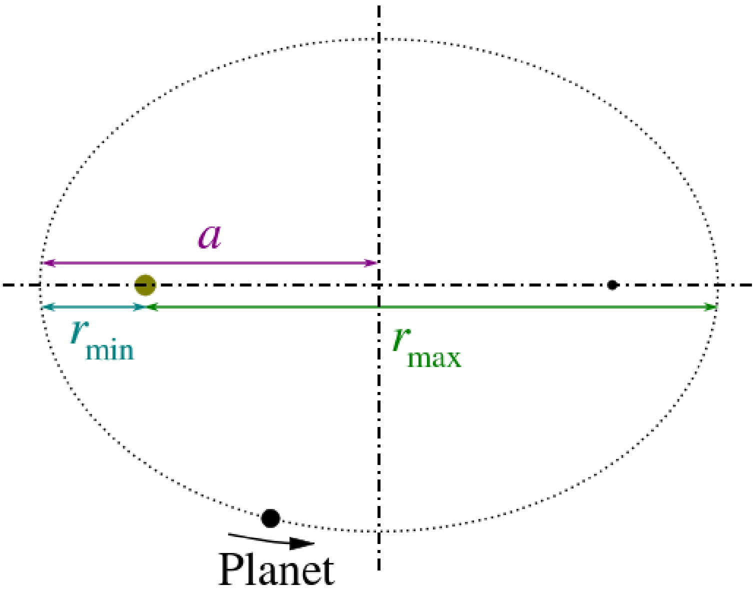
Kepler's second law states that a planet moves in its ellipse so that the line between it and the Sun placed at a focus sweeps out equal areas at equal times. (see animated gif: https://commons.wikimedia.org/wiki/File:Kepler-second-law.gif)



Author: Talifero; Source:https://commons.wikimedia.org/wiki/File:Kepler%27s_law_2_en.svg [Requested on January the 4th of 2022]

1.4.2.3. The third Kepler's law.

The square of a planet's orbital period is proportional to the cube of the length of the semi-major axis of its orbit.



Author:Stündle; Source:https://commons.wikimedia.org/wiki/File:Kepler_third_law_diagram.svg [Requested on January the 4th of 2022]

This law can be mathematically expressed in different and equivalent ways:

$$T^2 = k \cdot a^3$$

$$T^2 = k \cdot r^3;\ r = \frac{r_{min} + r_{max}}{2}$$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

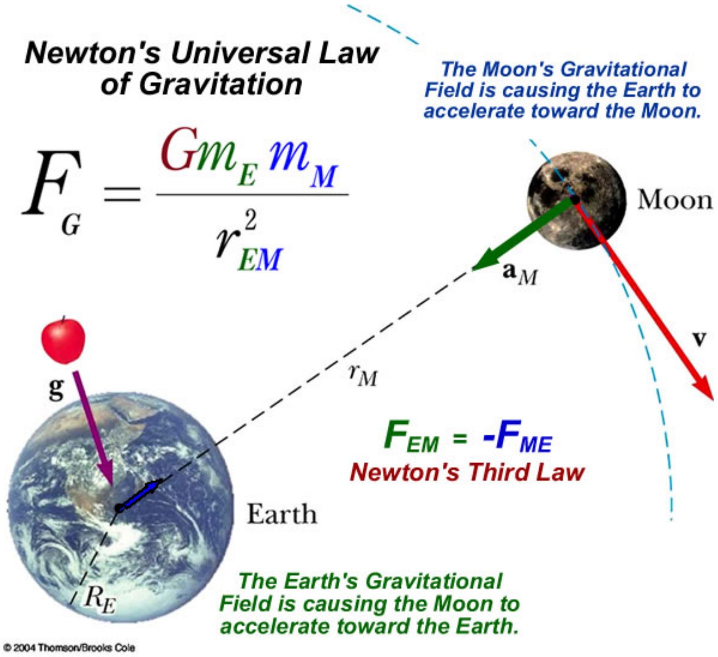
You can check this expression by replacing on it some attributes of the solar system planets: https://en.wikipedia.org/wiki/Planet.

If you want to learn more about Kepler's law from an historical point of view, you can watch this video: Episode 21: Kepler's Three Laws - The Mechanical Universe.

2. Gravitational Newton's law.

2.1. Mathematical expression.

What this law points out is that the force causing an apple to fall and the Moon to move around the Earth is the same.



Author:Thomson/Brookes Cole; Source:https://sdsu-physics.org/physics180/physics195/Topics/chapter13.html [Requested on January the 4th of 2022]

2.2. Deduction process.

This law was deduced assuming that the motion of the Moon around the Earth can be approached as a circular one and therefore be considered as a UCM and taking profit of Kepler's third law as well as his own Newton's third law.

- From the point of view of the Earth: whereas M stands for the mass of the Earth in this case.

$$F = M \cdot a_n$$

$$F = M \cdot \frac{v^2}{r}$$

$$F = M \cdot \frac{(\omega \cdot r)^2}{r}$$

$$F = M \cdot \omega^2 \cdot r$$

$$F = m \cdot \left(\frac{2\pi}{T}\right)^2 \cdot r$$

$$F = \frac{4\pi^2 M \cdot r}{T^2}$$

$$F = \frac{4\pi^2 \cdot M \cdot r}{k \cdot r^3} = \frac{4\pi^2 M}{k \cdot r^2}$$

- From the point of view of the Moon: the whole process would be equivalent but just setting the Moon in the center of a circle along which it would seem the Earth to be moving, whereas m stands for the mass of the Moon in this case. So, we will obtain the same result as in the previous point just replacing M by m:

$$F = \frac{4\pi^{2} m}{k \cdot r^{2}}$$

- If we just merge both solution and put together all constants in a unique constant that we can call G, the final result for the universal gravitational force (in module) is:

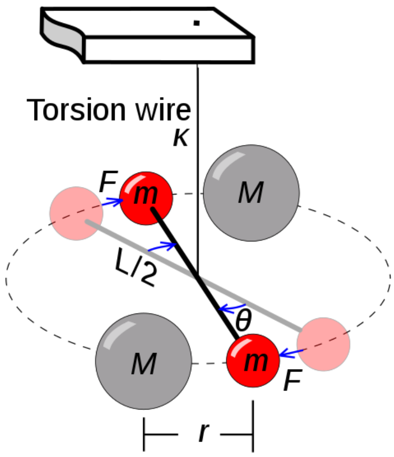
$$F = G\frac{M \cdot m}{\cdot r^{2}}$$

2.3. Experimental verification: Cavendish's experiment.

About 150 years later, Cavendish could confirm the validity of Newton's gravitational law by performing an experiment from which the value of G and, from there, the mass of the Earth could be determined.

You can see these videos to figure out how this experiment was done:

- Cavendish Experiment.
- Determining Gravitational Constant.
- Gravitational Attraction.



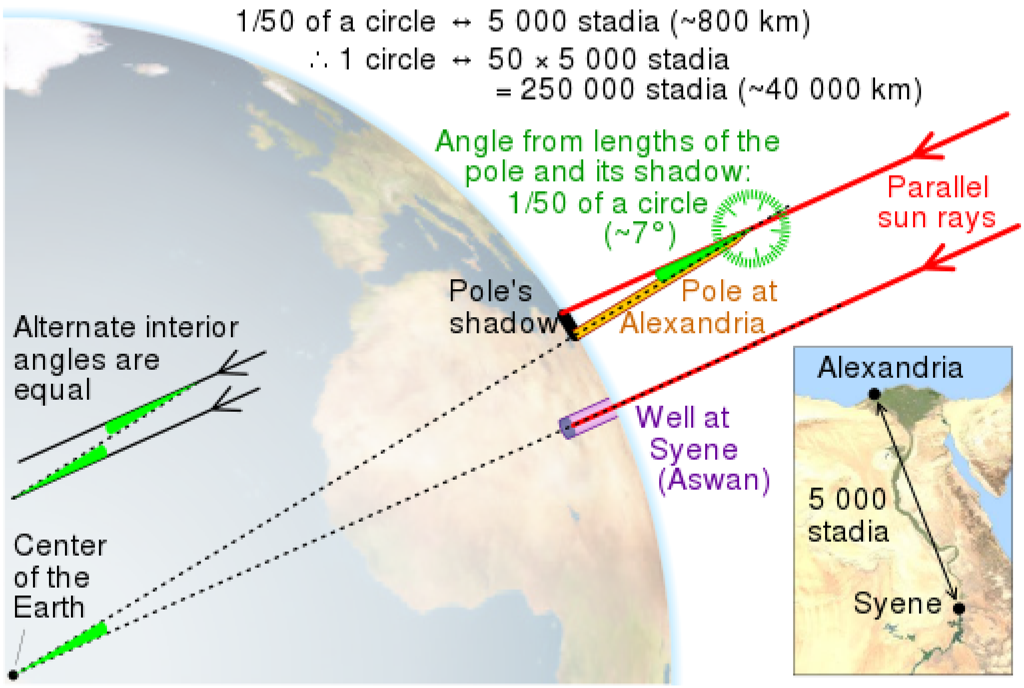
Author:Chris Burks (Chetvorno); Source:https://commons.wikimedia.org/wiki/File:Cavendish_Torsion_Balance_Diagram.svg [Requested on January the 5th of 2022]

$$G \approx 6.67428 \times 10^{-11}\, m^{3}\, kg^{-1}\, s^{-2}$$

If you want to learn more about Newton's universal gravitational law from an historical point of view, you can watch this video: Episode 8: The Apple And The Moon - The Mechanical Universe.

2.3.1. Eratosthenes and the measure of the Earth radius.

To measure the radius of the Earth, Eratosthenes measured the shadow cast by a mast during the summer solstice, just before noon, in the city of Alexandria, famous for the large library that burned there, and which is north of the Tropic of Cancer, while in the city of Siena, south of the tropics, a mast did not cast any shadow.



Author:cmglee, David Monniaux, jimht at shaw dot ca;
Source:https://simple.wikipedia.org/wiki/File:Eratosthenes_measure_of_Earth_circumference.svg [Requested on January the 5th of 2022]

More details about how Earth's radius was determined by Eratosthenes can be found in these pages: (R = 6370 Km)

- https://www.britannica.com/biography/Eratosthenes/images-videos.
- http://www.geo.hunter.cuny.edu/~jochen/gtech201/lectures/lec6concepts/datums/determining%20the%20earths%20size.htm.

2.3.2. Determination of the Earth mass.

Once Cavendish determined the value of G from his famous experiment, the determination of Earth's mass (M) was direct. Because we know the value of the acceleration of the gravity on Earth's surface:

$$F_g = G\cdot\frac{M\cdot m}{R^2} = m\cdot a = m\cdot g$$

$$M = \frac{g\cdot R^2}{G} = 5,997\cdot 10^{24}\, Kg \simeq 6\cdot 10^{24}\, Kg$$

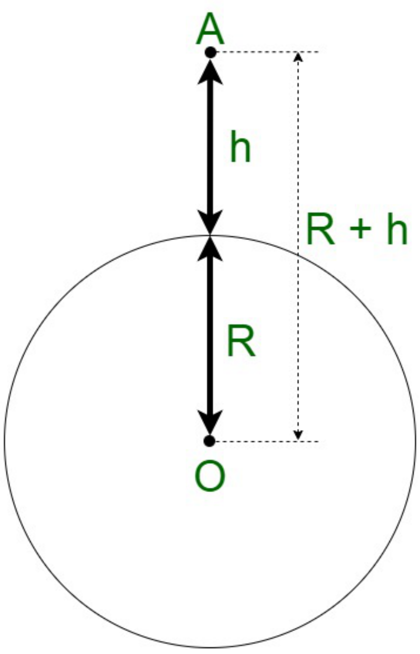
2.4. Variation of the acceleration of the gravity with height.

As you can see in the previous subsection the value of the acceleration of the gravity of 9,8 m/s2 is just constant indeed on Earth's surface and by the way it's equal to:

$$g = G \cdot \frac{M}{R^{2}}$$

This means that if an object is at a considerable high (k) in comparison with Earth's radius the value of g will decrease and we we will be able to calculate its value as:

$$g = G \cdot \frac{M}{(R+h)^{2}}$$



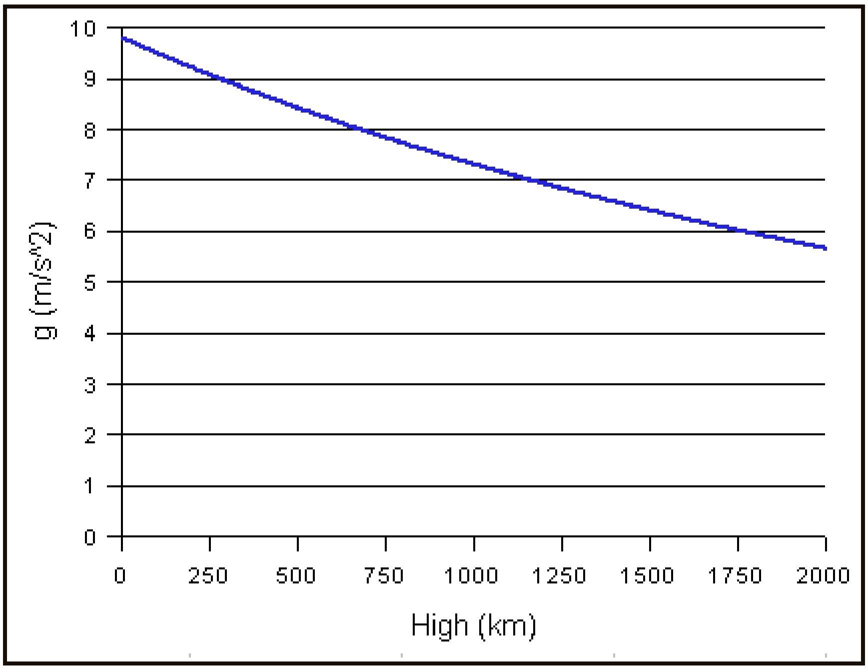


Image on the left: Author: yashchuahan; Source: https://www.geeksforgeeks.org/variation-in-acceleration-due-to-gravity/
Image on the right: Author: Dantor; Source: https://commons.wikimedia.org/wiki/File:Erdgvarp.png
[Both requested on January the 5th of 2022]

If you want to review Kepler's laws as well as Newton's universal gravitational law **in Valencian** you can check this link:

- https://fisquimcomval.wordpress.com/ley-de-gravitacion-universal/

If you want to review the weight force in relation with gravity and the variation of g with height in Valencian you can check this link:

- https://fisquimcomval.wordpress.com/fuerza-peso-la-caida-de-los-cuerpos-y-el-movimiento-orbital/

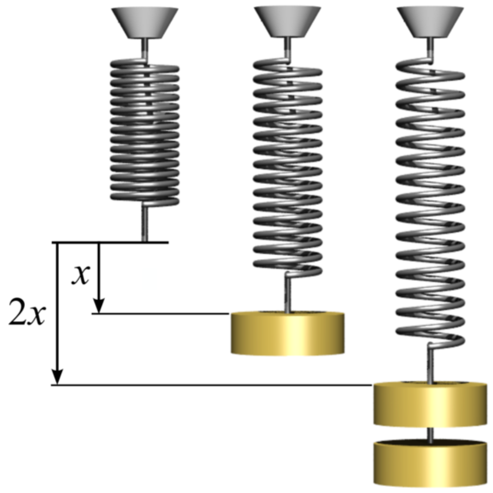
3. Hooke 's law.

As you saw in Cavendish's experiment a kind of elastic spring (a circular one in that case) was used to determine the force produced by the attractions between the masses. How does this kind of elastic spring behave? How can we describe the force appearing on them when they are contracted or extended? We will study it in case the springs are linear.

By the way, the elastic restoring force of the springs was studied by Robert Hooke, an English scientist contemporary with Newton and with whom he had a great confrontation throughout his life[3].

When you apply a force to a spring, it will probably get longer. If you duplicate the strength, the elongation will also double. This is what is known as Hooke's law.

$$F = -k \cdot (x - x_0)$$

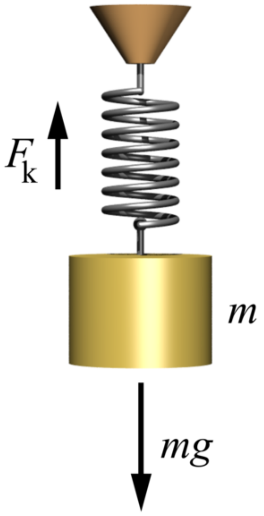


Author:Svjo; Source:https://commons.wikimedia.org/wiki/File:Hookes-law-springs.png [Requested on January the 5th of 2022]

whereas:

- F is the module of the force applied to the spring.
- k is the elastic constant of the spring, which relates force and elongation. The higher its value, the more work it will cost to stretch the spring. Depends on the type, and for each spring it is different.
- x_0 es la longitud del muelle sin aplicar la fuerza o longitud inicial.
- x es la longitud que se estira el muelle al aplicar la fuerza.

In this last image we can appreciate that for a vertical spring to be equilibrium the elastic restoring force and the weight must be equal in module while they point out to opposite directions.



Author:Svjo; Source:https://commons.wikimedia.org/wiki/File:Mass-spring-system.png [Requested on January the 5th of 2022]

[3] Both belonged to the Royal Society of London and both achieved the role of president of it at different times.