LESSON 1: KINEMATICS I – THE STUDY OF MOTION

1. Reference System.

1.1. What is relative motion and why we need to talk about inertial reference frames.

Watch this video to think about the importance of having a reference system to describe any kind of motion relative to it: shorturl.at/bevzD.

Relative Motion and Inertial Reference Frames
395.218 visualizaciones • 10 feb 2017

👍 7135   👎 237   ➔ COMPARTIR   ≡+ GUARDAR   •••

Professor Dave Explains ✓
1,45 M de suscriptores   UNIRME   SUSCRIBIRME

1.2. Location depends on the frame or system of reference.

Think about that. You are sitting at a given place in the classroom, right? So, imagine you have to indicate to someone where are you sitting right now. What kind of information do you need to provide for someone to figure out where are you sited in the simplest possible way?

Ok, I hope you have realized that you will need two things:
- Define a **reference frame/reference system**, from where you can
- **Measure** the **distance**/s you stay from in such a way that anyone else can use it to exactly know where you are.

It is not the same to measure the middle point of a given table from the corner close to the professor's table than to do it from the one close to the main entrance door.

1.3. Cartesian system of reference.

In Physics we use a Cartesian system of reference with axis x and y to locate a dot in a plane.
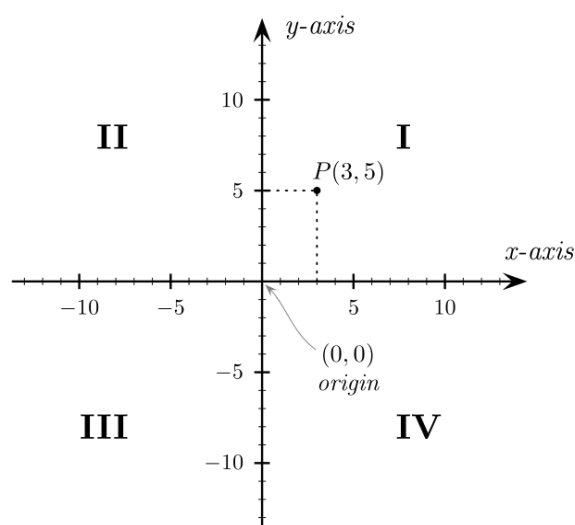
As you can observe, we need to provide two measures in order to locate the point P relative to the origin of our reference system: 3 and 5 (let's figure out they stand in meters).
- The 3 represents the distance to the origin of the projection of P to the x-axis (imagine you illuminate P just from above and register where its shadow falls on the x-axis).
- The 5 represents the distance to the origin of the projection of P to the y-axis (imagine you illuminate P just from the right and register where its shadow falls on the y-axis).
- We combine these two values following this notation: (3, 5) m.

You can review the system of reference **in valencian** from this educational blog link:
https://fisquimcomval.wordpress.com/2020/09/01/sistema-de-referencia/

1.4. Unit vectors in Cartesian coordinates.

It is also very common to use unit vectors as a basis from which to locate a point relative to the origin of a reference frame. Each of these unit vectors matches with one axis and is oriented from the origin to the direction in which the values increase along this axis, which is very intuitive because it ends in a narrow pointing towards that direction. As their name indicates these vectors have all of them a value equal to one (in reality as value we mean the module of the vector; you will learn what this is in section 4.2).
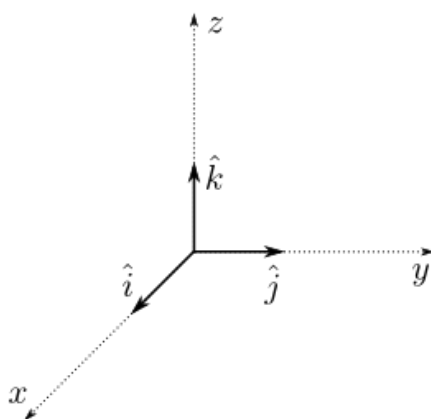
So, using vectors, and based on the unit vectors along the axis x and y, we can refer to the point P that we have seen in the previous subsection, alternatively as a vector position, as we are going to see in section 2.

2. Position vector.

It indicates the position of a particle or any other object we can fit in a point, relative to the origin of a reference frame.

The symbol that is more commonly used to refer to it is the letter 'r' with a narrow on it, which expresses the fact that this is a vector, not just a number.

For instance, the point P that we have seen in the subsection 1.3 can be pointed out by the vector position $(\vec{r})$, which can be written in two possible ways, as, $\vec{r} = (3,\ 5) = 3\,\hat{i} + 5\hat{j}$, followed by the proper units that correspond to these values.
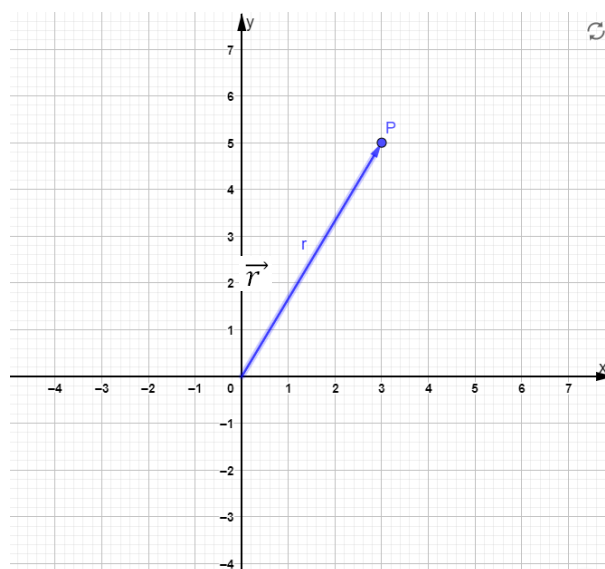
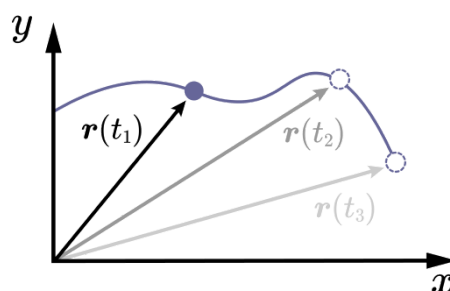$$\vec{r} = (3,\ 5)\,m = \left(3\,\hat{i} + 5\hat{j}\right)m$$

So, in a more generic way, we can say that the vector position can be expressed like

$$\vec{r} = (x,\ y) = \ x\,\hat{i}\ +\ y\hat{j}$$

Again, this expression should be followed by the proper units.

In practical terms, in physics, this vector will change with time. So, for each and every time we will have a given vector position with their given components and representation.

In the international system the position vector is measured in meters (m).

3. Trajectory.

The trajectory corresponds to the line defined by the "bunch of points" through which the particle has passed. This usually defines a curved line, as the one we can see in the previous image in a color between cyan and violet. The trajectory corresponds with the real traveled space.

4. Displacement.

The trajectory has to do with the **distance** associated with the **space traveled** on depending on the path taken. But another important concept is the **displacement**, which has to do with the distance between the start and the end point.
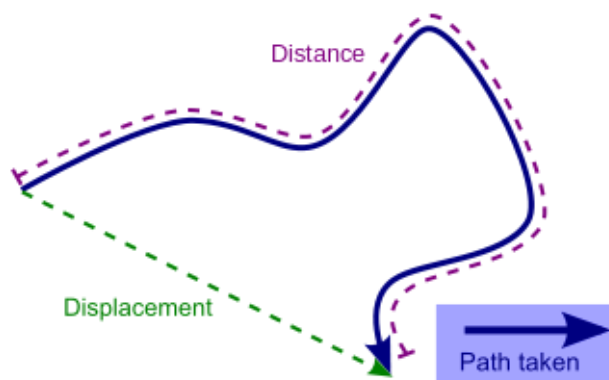
In the next image you can see an example that can be used to understand how to calculate both the distance based on the path taken and the displacement.
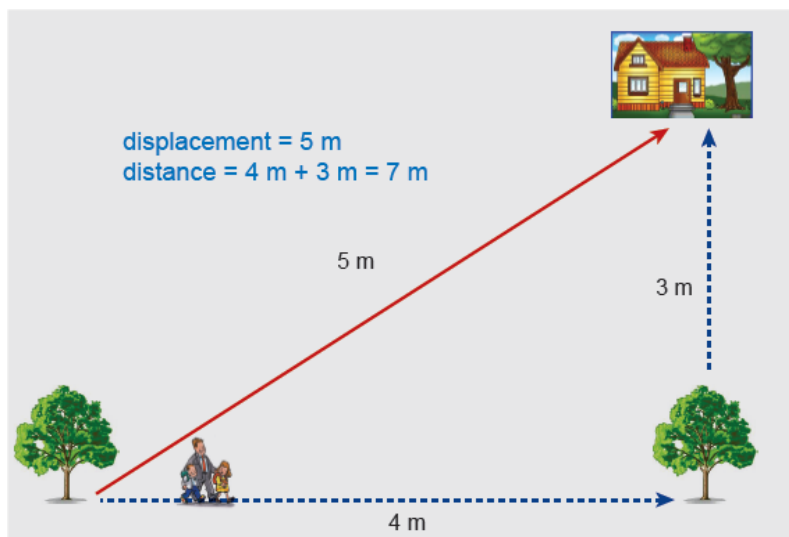
In this image the dashed blue line ending with a narrow indicates the path taken by the father and his children, and it can be used to calculate the distance that they really have walked. While, in opposition, the red line ended with a narrow that represents the displacement. Do you realize that the two branches of the path taken and the one of the displacement are set in this illustration as the three sides of a rectangle triangle? So, what should you do to calculate the displacement in one case like that if you would just know the length of the two sides corresponding to the path taken?

You have to use the Pythagorean Theorem!!! Do you remember it from Math???
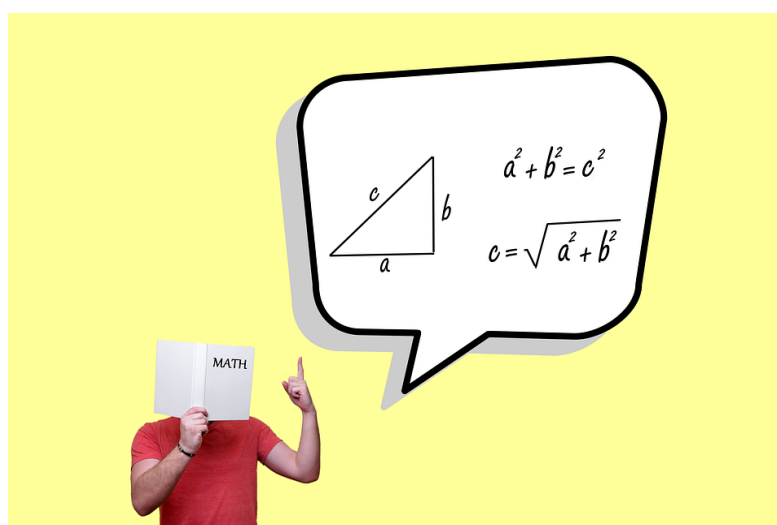
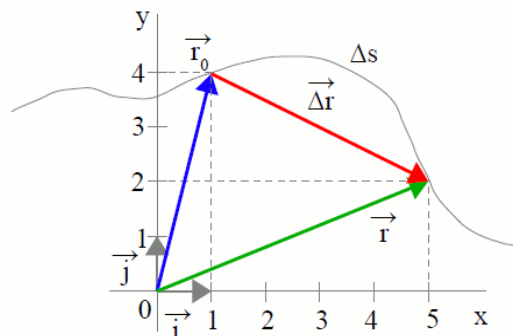So, you can check and verify that indeed $\sqrt{3^2 + 4^2} = 5\ m$.

4.1. Displacement vector.

The displacement has to be handled as a vector because, apart from a length, it must include direction and orientation. The displacement vector is usually designed as $\overrightarrow{\Delta r}$. And, as you can see, it is drawn as a vector going from the end of the initial position vector narrow to the end of the final position vector's narrow, and, of course, also ended by a narrow.
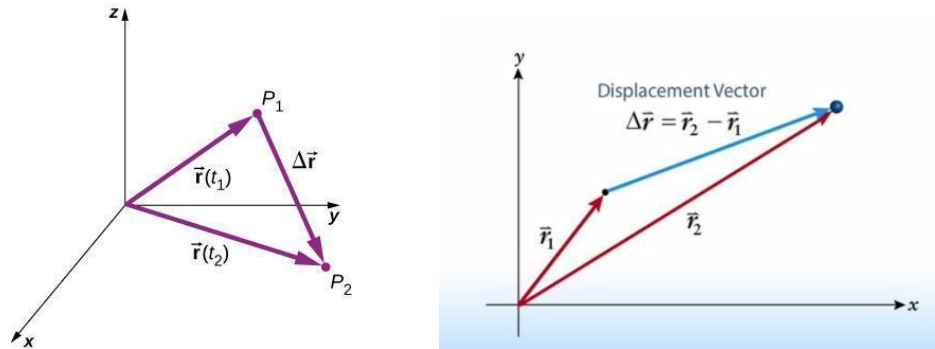
Author: Beta Keramati; Source: https://introphys1.pressbooks.com/chapter/unit-4-vectors-and-kinematics/ [Requested on August the 7th of 2021]

In the image on the left you can see how to draw the displacement vector in space, so, in three dimensions. In the image on the right, you can see how to do it in two dimensions. In both cases, you get it drawn with a line going "from the initial to the end position vector"; so, from $\overrightarrow{r}_1$ to $\overrightarrow{r}_2$, although many often the initial position vector is written as $\overrightarrow{r}_0$ while the final position vector is as just $\overrightarrow{r}$.

Author: https://www.alonsoformula.com/ ; Source: https://www.alonsoformula.com/FQBACH/cinematica.htm

You can get a better feeling about the difference between the distance associated with the path taken and the displacement using this link: http://www.educaplus.org/game/distancia-y-desplazamiento.

The question now is how can the displacement vector be calculated from the start and end position vectors. We will face it supported in the last image.

$\overrightarrow{\Delta r} = \overrightarrow{r} - \overrightarrow{r}_0$, whereas

$\overrightarrow{r}_0 = (1, 4) = \hat{i} + 4\hat{j}$ and

$$\vec{r} = (5, 2) = 5\hat{i} + 2\hat{j}$$

So, vector subtraction is done component by component: $\vec{\Delta r} = (5 - 1, 2 - 4) = (4, -2)m$

4.2. How to calculate the module (length) of a vector.

It is very often a matter of interest to calculate the length of a vector, what it is also called as his module. Essentially, in order to calculate the length of a vector, the Pythagorean Theorem has to be applied. Let's do that by following a couple of examples.

Example 1: calculate the length of the vector drawn in section 2, page 3.

So, let's write the vector based on their components: $\vec{r} = (3, 5)m = \left(3\hat{i} + 5\hat{j}\right)m$
Now, applying the Pythagorean Theorem we will calculate its length or module, which can be expressed as:

$$r = \left|\vec{r}\right| = \sqrt{3^2 + 5^2} = 5,83 \; m$$

Example 2: calculate the length of the displacement vector drawn in subsection 4.1, page 6.
In that case, we will express the process of calculation in a most generic way possible and then apply the specific values extracted from the mentioned case.

$$\vec{r}_0 = \left(x_0, y_0\right) = (1, 4)m = \left(\hat{i} + 4\hat{j}\right)m$$
$$\vec{r} = (x, y) = (5, 2)m = \left(5\hat{i} + 2\hat{j}\right)m$$
$$\vec{\Delta r} = \left(x - x_0, y - y_0\right) = (5 - 1, 2 - 4) = (4, -2) \; m$$
$$\Delta r = \left|\vec{\Delta r}\right| = \sqrt{(x - x_0)^2 + (y - y_0)^2} = \sqrt{4^2 + (-2)^2} = 4,47 \; m$$

So, as a summary, take note of it:

$$\vec{r}_0 = \left(x_0, y_0\right); \; \vec{r} = (x, y)$$

$$r = \left|\vec{r}\right| = \sqrt{x^2 + y^2}$$

$$\vec{\Delta r} = \left(x - x_0, y - y_0\right)$$

$$\Delta r = \left|\vec{\Delta r}\right| = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$

Of course, in general, the displacement ($\vec{\Delta r}$) doesn't matches with the trajectory (Δs). But both of them are measured in meters (m) in the International System of Units.

5. Speed.

In general, the speed is a vector magnitude that indicates how the position of a mobile varies with time. The units of speed in the International System of Units are [m/s]. Now, **think about that**: as far as it is a vector magnitude, the speed can change due to a change in its module, but also due to a change in its direction.

5.1. Average speed.

The average speed is defined from two specific points of the trajectory associated to their own corresponding position vectors and time instants.
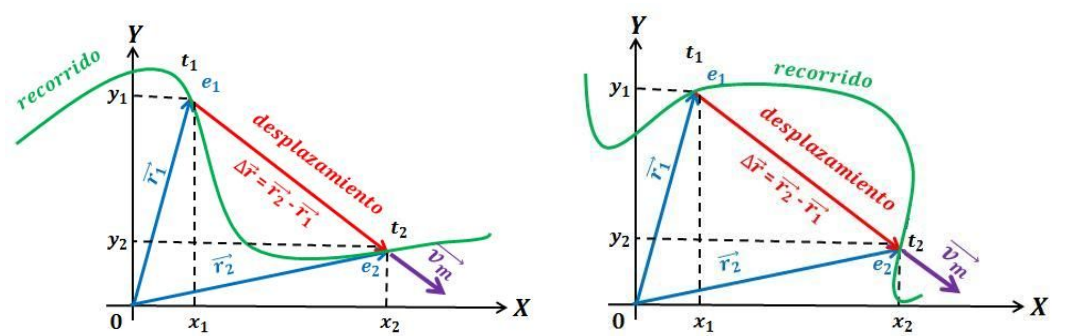
As you can see, as far as the average speed only depends on two points, it doesn't depend on the traveled space associated to the trajectory, but to the displacement vector divided by the time lapse.

$$\vec{v_m} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r_2} - \vec{r_1}}{t_2 - t_1}$$

In opposition to the average speed is the definition of the average celerity, which in this case is indeed defined by the space traveled between the two points and, because of that, it does depend on the trajectory.

$$c_m = \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1}$$

Which kind of trajectory will guarantee the average speed to be equal to the average celerity? You can check this webpage to review this concept: https://www.fisicalab.com/en/section/average-speed.

You can also access this Alfredo Bello's Geogebra page to play with points and times to figure out how position and average speed vectors depend on them: https://www.geogebra.org/m/vdsa7a7k.
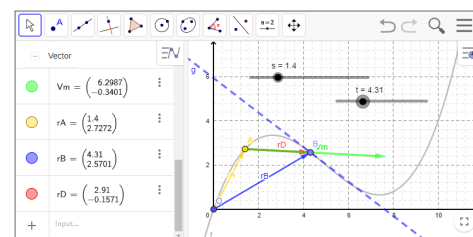
5.2. Instant speed.

The instant speed is the speed associated with each and every specific time. It can be thought of as the speed corresponding to a change in the vector position associated to change in time tending to zero[1].

You can review the concepts of vector position, space traveled, displacement and average and instant speed **in valencian** from this educational blog: https://fisquimcomval.wordpress.com/2020/09/01/magnituds-del-moviment-posicio-trajectoria-i-desplacament/

6. Acceleration.

The acceleration is defined as the change of the speed per unit of time. Again, in general, it is a vector magnitude. And their units in the International System are $[m/s^2]$.
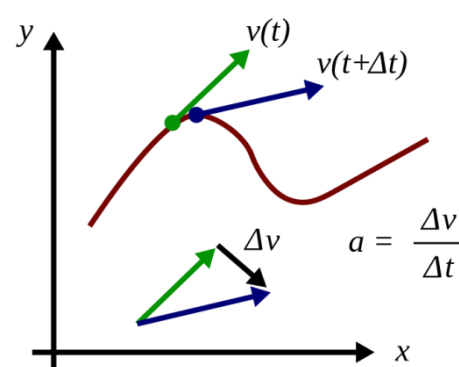
When the acceleration is calculated as the change in the speed between two points separated by a finite and measurable distance, we will talk about **average acceleration**.

You can check this webpage to review this concept: https://www.fisicalab.com/en/section/average-acceleration

As the speed can change in both module and direction, we will see how, on depending on that, two kinds of acceleration can be defined:
- Tangential acceleration.
- Normal acceleration.

But we will introduce these two kinds of acceleration, for a better understanding of them, after we have studied the circular motion.

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[1] This is the concept we will resume in the next course as to deal with instant speed.

7. Uniform rectilinear motion (URM).

This kind of movement is characterized by two facts:

1. The **speed** remains always constant on its value (module). This implies the average and the instant speed to be equal since its value doesn't change.
2. The **speed** remains also constant on its direction. This implies the object to be always moving along a straight line.
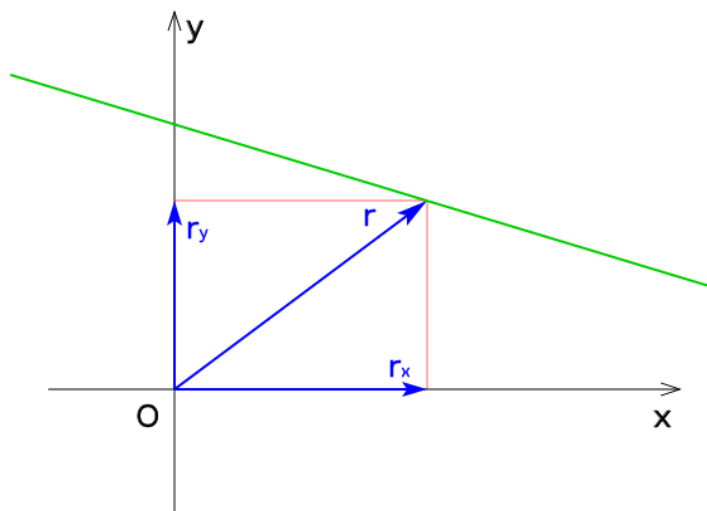
Author: Dnu72; Source: https://commons.wikimedia.org/wiki/File:Posici%C3%B3n_2D_recta.svg [Requested on August the 7th of 2021]

As far as the speed is constant by definition in this kind of motion, it will, in particular, be always equal to the average speed and, because the mobile moves always along a straight line, the simplest way to define it is choosing this x-axis as to match with it. Taking these two ideas into account we can derive the equation of the uniform linear motion as follows:

$$v = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0}$$

Whereas x_0 stands for the initial position, corresponding to the initial time t_0 (very often, $t_0 = 0$, especially when we study the motion of just one mobile).

From the relations above, we can derivate:
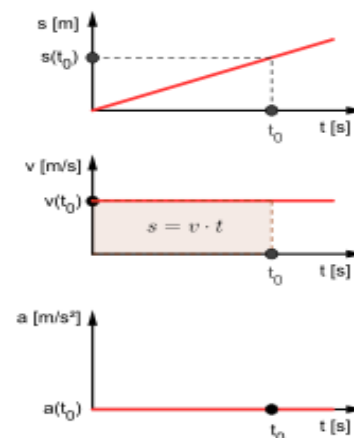
$$x - x_0 = v \cdot (t - t_0)$$
$$x = x_0 + v \cdot (t - t_0)$$

For the specific case of $t_0 = 0$, it results: $x = x_0 + v \cdot t$, which is the more common and simplest form of the uniform rectilinear motion equation, although the most generic one must include the t_0 term (especially for those cases of study with two mobiles starting their motions at different times).

7.1. Graphs of uniform rectilinear motion.

This kind of motion is characterized by these 3 different graphs:

- The first graph represents the dependence between space (in that case represented by s instead of x) and time.
    - o If the speed is positive ($v > 0$) the slope of the line is positive and so the position increases with time.
    - o If the speed is negative ($v < 0$) the slope of the line is negative and so the position decreases with time.

- The second graph represents the dependence between speed and time. As you can see, the speed just remains constant. <u>It is interesting to highlight that the area defined by the rectangle surrounded by the line and a given time lapse is equal to the space travelled</u>.

- The third graph just points out the fact of the acceleration to be null all the time.

You can review the URM graphics from these pages:
- ▪ https://www.fisicalab.com/en/section/urm-graphics.
- ▪ https://www.phyley.com/uniform-linear-motion.
- ▪ https://ekuatio.com/en/uniform-rectilinear-motion-solving-problems-step-by-step/.

You can review the concepts of vector position, space traveled, displacement and average as well as instant speed **in Valencian** from this educational blog post:
https://fisquimcomval.wordpress.com/2020/09/01/moviment-rectilini-uniforme-mru/

You can review the steps for the resolutions of problems based on URM **in Valencian** from this post:
https://fisquimcomval.wordpress.com/2020/09/01/estrategies-per-a-la-resolucio-de-problemes-de-cinematica/

8. Uniform accelerated rectilinear motion (UARM).

This kind of movement is characterized by two facts:
1. The **acceleration** remains always constant on its value (module). This implies the speed to change constantly.
    - ▪ It increases constantly if the acceleration is positive (a > 0).
    - ▪ It decreases constantly if the acceleration is negative (a < 0).
2. The **speed** remains also constant on its direction. This implies the object to be always moving along a straight line.

As far as the acceleration is constant by definition in this kind of motion, it will, in particular, be always equal to the average acceleration and, as the mobile moves always along a straight line, the simplest way to define it is choosing this x-axis as to match with it. Taking these two ideas into account we can derivate the equation of the uniform linear motion as follows:

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0}$$

Whereas v_0 stands for the initial speed, corresponding to the initial time t_0 (very often, $t_0 = 0$, specially when we study the motion of just one mobile).

From the relations above, we can derivate:

$$v - v_0 = a \cdot (t - t_0)$$
$$v = v_0 + a \cdot (t - t_0) = v_0 + a \cdot \Delta t \ (1)$$

For the specific case of $t_0 = 0$, it results: $v = v_0 + a \cdot t$, which is the more common and simplest form of the first uniform accelerated rectilinear motion equation, although the most generic one must include the t_0 term (especially for those case of study with two mobiles starting their motions at different times).

But in this kind of motion there are two more generic equations we should derivate for a complete definition of it. In order to derivate the second equation of the UARM we must first study the form of the graphs associated to it.

8.1. Graphs of uniform accelerated rectilinear motion.

This kind of motion is characterized by these 3 kinds of graphs:

- The first graph represents the dependence between space and time. In the next page we will see how to derivate the equation that expresses the dependence between x and t.

- The second graph represents the dependence between speed and time. As you can see, the speed just increases (or decreases) constantly. It is interesting to highlight that the area defined by the line and a given time lapse is still equal to the space travelled (as we will resume in the next page).
  - If the acceleration is positive ($a > 0$) the slope of the line is positive as well as for and so the speed increases with time.
  - If the acceleration is negative ($a < 0$) the slope of the line is negative and so the speed decreases with time.

- The third graph just points out the fact of the acceleration to be null all the time.
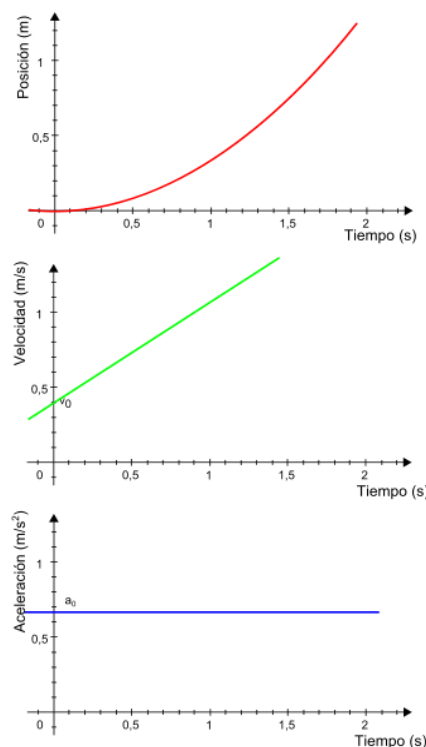
Author: Robot8A; Source: https://commons.wikimedia.org/wiki/File:Grafico_pva_del_MRUA.svg [Requested on August the 18th of 2021]

8.2. Derivation of the second equation of the UARM.

Let's now focus on the second graph, the one correlating speed and time in a UARM.
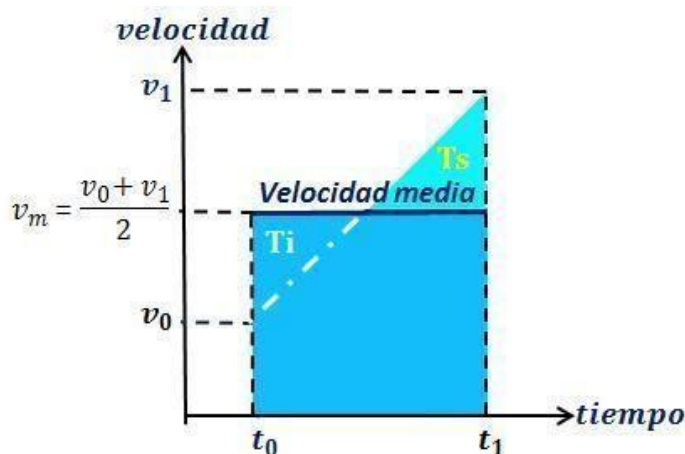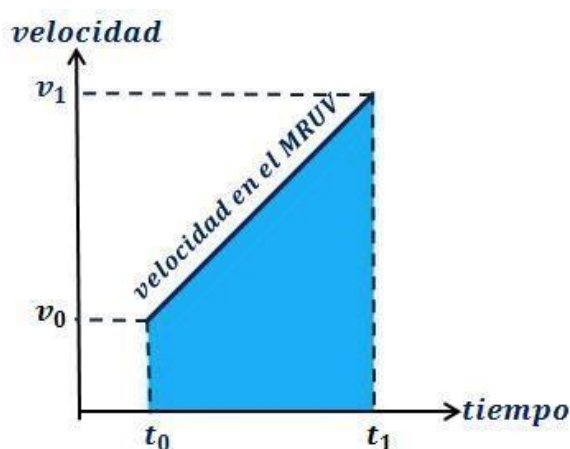
8.2.1. Calculation of the average speed.

A useful step to do in order to calculate the second equation of the UARM is to determine the average speed in the time range from t_0 to t_1. If we suppose the initial speed to be $v_0 = 10 \, m/s$ and the final speed to be $v_1 = 20 \, m/s$, it is pretty much intuitive that the calculation of the average speed will have to be as follows:

$$v_m = \frac{v_0 + v_1}{2} = \frac{10+20}{2} = \frac{30}{2} = 15 \, m/s.$$

8.2.2. Calculation of the space traveled.

The advantage of having calculated the average speed is that now you can redraw the graph including it on it. Now, the average speed can be though as the constant speed an URM would have to have in order the travel the same space in the same time lapse. So, coming back to the idea introduced and underlined in section 7.1 we can express how to calculate the space traveled as the area surrounded by the blue rectangle (in one dimension).

$$\Delta x = x - x_o = v_m \cdot \Delta t = \frac{v_0 + v_1}{2} \cdot (t_1 - t_0)$$

On the other hand, we can make use of the first equation of the UARM that we ha derivate in section 8 to express $v_1 = v_0 + a \cdot (t_1 - t_0)$ in order to replace v_1 by it in the relation above.

$$x = x_o + \frac{v_0 + v_0 + a \cdot (t_1 - t_0)}{2} \cdot (t_1 - t_0)$$

$$x = x_o + v_0 \cdot (t_1 - t_0) + \frac{1}{2} a \cdot (t_1 - t_0)^2$$

For a more generic, undefined and variable final time, this expression is usually written as:

$$x = x_o + v_0 \cdot (t - t_0) + \frac{1}{2} a \cdot (t - t_0)^2$$

Or even alternatively:

$$x = x_o + v_0 \cdot \Delta t + \frac{1}{2} a \cdot \Delta t^2 \ (2)$$

It is also very common to write this equation in a simplest way for when the initial time can be assigned to zero ($t_0 = 0 \ s.$).

$$x = x_o + v_0 \cdot t + \frac{1}{2} a \cdot t^2$$

8.3. Calculate traveled space from areas.

If you look again at the image from section 8.2 we will be able to realize that the area defined by the rectangle on the right is equal to the one defined by the trapezoid on the left. If you pay attention, you will see that this is so because it is like you have cut a triangle from the upper right corner of the trapezoid (Ts) and you have added it to the upper left side after rotating it 180 degrees to the left (Ti).

In other words, <u>you can always calculate the space traveled directly as the area limited by the graph itself of the speed versus the time in a given time lapse</u>[2]. At the end, this trick can be applied always, regardless of the form that the speed versus time graph has.

You can review how to calculate traveled space from areas and how to apply it to derivate the second equation of an UARM in the following pages:

- https://www.fisicalab.com/en/section/uarm-graphics
- http://www.hk-phy.org/contextual/mechanics/kin/mo_gr02_e.html
- https://www.physicsclassroom.com/calcpad/1dkin

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[2] Although the space traveled is calculated from an area, the units are not the proper of an area, like for example would be quadrat meters, but the ones resulting of the product between the magnitudes in the resulting rectangle that are associated to the base and the height: speed x time = (m/s) x s = m, in the International System of Units.

8.4. Derivation of the third equation of the UARM.

The third and last equation of the UARM emerges as the combination of the two others. We just have to isolate Δt in the first equation and replace the resulting expression in the second one, as to obtain an equation that doesn't depend on time.

$$\Delta t = \frac{v - v_0}{a}$$

$$x = x_o + v_0 \cdot \left(\frac{v - v_0}{a}\right) + \frac{1}{2} a \cdot \left(\frac{v - v_0}{a}\right)^2$$

$$\left(x - x_o\right) \cdot a = v_0 \cdot v - v_0^2 + \frac{1}{2}\left(v^2 - 2v \cdot v_0 + v_0^2\right)$$

$$2a \cdot \left(x - x_o\right) = 2v_0 \cdot v - 2v_0^2 + v^2 - 2v \cdot v_0 + v_0^2$$

$$v^2 - v_0^2 = 2a \cdot \Delta x \quad (3)$$

You can review the steps for the resolutions of problems based on UARM **in Valencian** from this post:
https://fisquimcomval.wordpress.com/2020/09/01/moviment-rectilini-uniformement-variat/

9. Free fall.
9.1. Galileo and the measurement of the gravity acceleration on Earth's surface.
9.1.1. Galileo and the study of the pendulum.

Galileo was the first scientific in introducing the experimentation and the collection and analysis of data in Europe after centuries dominated by the dogmatism based on ideas no validated[3]. All started when a very young Galileo went to the cathedral of Pisa and observed the movement of the lamp.

Galileo asked himself questions like:
- Is the time required to travel back and forth depending on the length of the string?
- Does it depend on the amplitude of the movement?
- Does it depend on the lamp mass?

Author: JoJan; Source: https://commons.wikimedia.org/wiki/File:Pisa.Duomo.dome.Riminaldi01.jpg [Requested on August the 20th of 2021]

In order to get answers to those questions he build a very simple pendulum at home and studied it[4]

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[3] The scientist considered as the first applying the experimentation as a key in the process of investigation was Alhazen of Basora in Egipt about the year 1000 a.C.

[4] As you will be able to do if you choose to do the optional task though as to improve your results in the first trimester.

9.1.2. Galileo and the tower of Pisa.

Galileo realized that the lamp of Pisa's cathedral moves because it falls. But the tension of the string is the cause for the lamp to move as a pendulum.

In order to study how objects fall without being fixed by a string and taking advantage of having born in the city of Pisa, Galileo went to the top of its famous tower to study how objects with different sizes and weights fall.

Old idea   Galileo
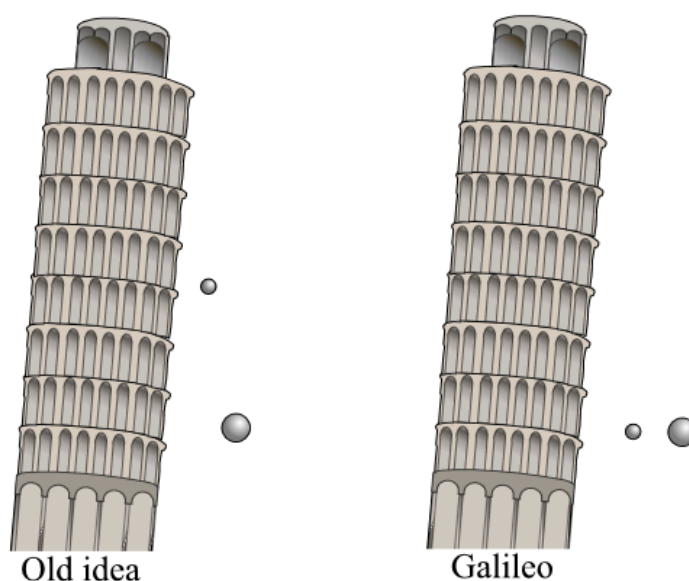
Author: QuiteUnusual; Source: https://commons.wikimedia.org/wiki/File:Pisa_experiment.png   [Requested on August the 20ᵗʰ of 2021]

What is labelled in the picture of the left as the old idea makes reference to the idea sustained by Aristotle and accepted during 14 centuries although it was never verified in an experimental way until Galileo did it to reject it. So, according to Aristotle, the weightiest body should fall faster a reach the floor sooner. The only thing is that Aristotle never verified his idea experimentally. But due to the extraordinary prestige of Aristotle his ideas endured for such a long time as unquestionable…until Galileo decided to face them. And what he found was amazing: two bodies with different sizes and masses fall all together if we let them free at the same time from a same height, also reaching the floor at the same time[5].

It is clear that all the objects must be accelerated from being attracted by the Earth in the same way, regardless of their mass, because the higher is the point from where they have a free fall, the stronger is the impact with the floor. So, Galileo wanted to determine this acceleration. But he found that the time required by the object he drops in a free fall from Pisa's tower was too short as to be able to measure them with the precarious media they have at this époque. So, he had to figure out a way to increase the

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[5] As far as they are not affected by the friction of the air. By the way, this idea was checked when the human being reached the moon, where there is no atmosphere, letting fall from a same height a hammer and feather, as it can be seen in this YouTube video, which title is "Hammer vs Feather - Physics on the Moon", from the channel AIRBOYD: https://www.youtube.com/watch?v=KDp1tiUsZw8&t=6s

time for an object to fall due to the attraction of the Earth that could allow, although it would be in an indirect way, to measure the acceleration of the gravity on it.

9.1.3. Galileo and the inclined plane.

Galileo realized that he could use a ball rolling down along an inclined plane as to increase the time required to reach the floor by the action of gravity, so as to measure the space traveled in equal time lapses, which could be measured by the oscillation of a pendulum.


Author: Archeologo (Museo Galileo); Source: https://commons.wikimedia.org/wiki/File:Piano_inclinato_inv_1041_IF_21341.jpg   [Requested on August the 20ᵗʰ of 2021]

A bunch of videos will help us to understand how Galileo proceeds to experimentally determine the acceleration of the gravity on the Earth's surface.

- Historical vision of the experiment of Galileo with a ramp:

- o Title: Galileo's Measure Of Gravity Explained By Jim Al-Khalili | The Amazing World Of Gravity | Spark
  - o YouTube Channel: Spark
  - o Link: https://www.youtube.com/watch?v=ZBr8Q2ROX9s
- Video animation of the experiment with a ramp from Galileo's museum:
  - o Link: https://catalogue.museogalileo.it/multimedia/InclinedPlane.html

In conclusion, Galileo found that the space traveled is proportional to the time squared.
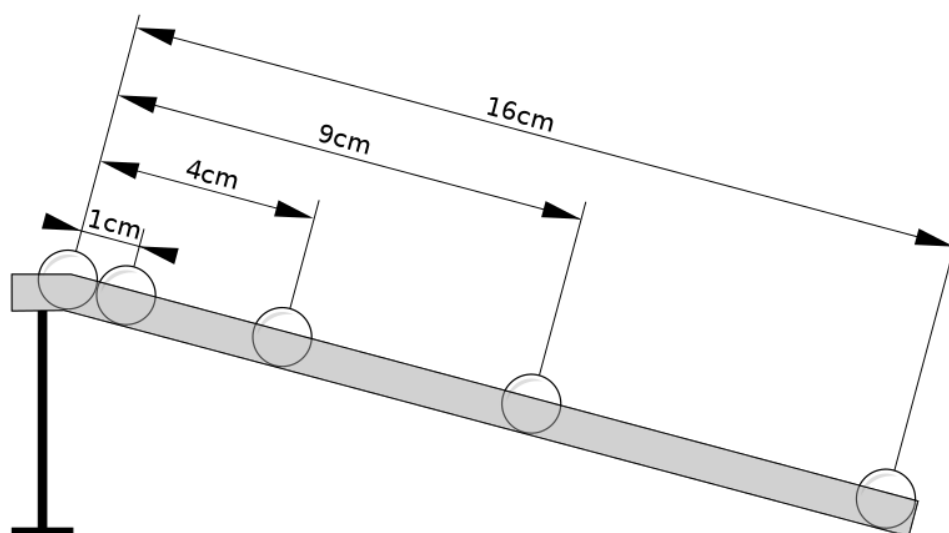
Author: MikeRun; Source: https://commons.wikimedia.org/wiki/File:Inclined-plane-experiment.svg [Requested on August the 20th of 2021]

- Video about how to collect data from a ball rolling down in a ramp with actual media:
  - o Title: Galileo's Kinematics Lab
  - o YouTube Channel: Alison Steward.
  - o Link: https://www.youtube.com/watch?v=7utvz1n4-X4
- Video about how to collect data from a ball rolling down in a ramp with actual media:
  - o Title: AP Lab Acceleration on an Inclined Plane
  - o YouTube channel: Sally Peppitoni.
  - o Link: https://www.youtube.com/watch?v=yHUbNxEjDIs
- Video about how to process data obtained from a ball rolling down in a ramp with actual media:
  - o Title: Acceleration down an inclined plane - graphing motion (NCPQ).
  - o YouTube channel: Richard Walding.
  - o Link: https://www.youtube.com/watch?v=2q_qzKT74Ow

9.1.4. Acceleration of the gravity on the Earth's surface.

The value corresponding to the acceleration of gravity on the Earth's surface can be determined indirectly from data taken from a pendulum or an inclined plane among other physical experiments and is results to be:

$$g = -9,8\,m/s^2$$

The negative sign expresses the fact that this acceleration is always attractive to the center of the Earth. In other words, if we align the y-axis to the vertical, the acceleration of the gravity increases in the opposite direction as the one in which this axis does.

9.1.5. Equations for the study of free fall motion.

A free falling motion is just a specific case of UARM. Then only things to do to obtain the equations that describe this kind of motion are:
- Replace x by y on the 3 equations.
- Replace 'a' by '–g' on them (in that case taking 'g' as positive since the minus sign is already included in the equation itself).
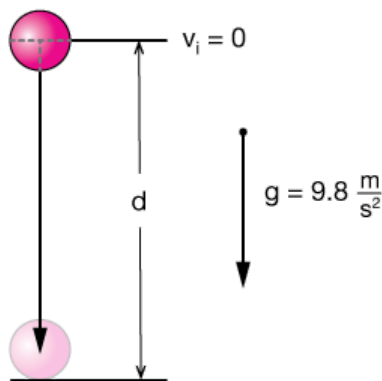

HORIZONTAL MOTION	FREE FALL
$v_f = v_i + at$	$v_f = v_i - gt$
$x_f - x_i = v_i t + \dfrac{1}{2}at^2$	$y_f - y_i = v_i t - \dfrac{1}{2}gt^2$
$v_f^2 = v_i^2 + 2a(x_f - x_i)$	$v_f^2 = v_i^2 - 2g(y_f - y_i)$

Author: Dr. Jeff Cruzan; Source: https://www.xaktly.com/Freefall.html [Requested on August the 19th of 2021]

At the time of solving free fall motion problems you will have to take into account two things:
- Signs:
  - Any magnitude associated with a vector pointing up will be positive.
  - Any magnitude associated with a vector pointing down will be negative.
- Conditions: you will always have to look for the way to "translate" the conditions related to what is asked to be found in mathematical terms. For example:
  - If you have to calculate the time required to reach the ground or the speed that it has when so, you will have to fix the final value of $y = 0$ (since the ground lies at a zero altitude in any system frame that makes sense).
  - If you have to calculate the highest altitude that an object reaches, you will have to fix the speed at that point $v_y = 0$ (since it just changes from having a positive sign while ascending to having a negative one while descending).
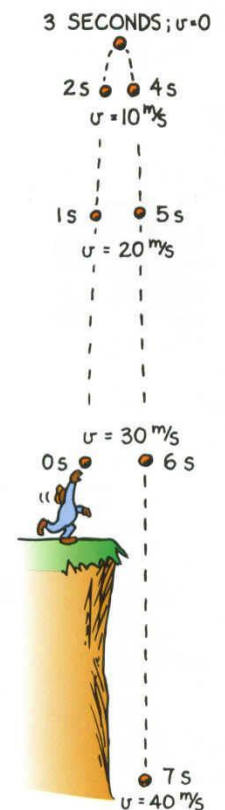
Author: Duncan Willson; Source: http://sctritonscience.com/Wilson/physics_free_fall_acceleration.htm [Requested on August the 19th of 2021]

In case two objects are moving in a given problem, then eventually you should use the same equations but replacing t by Δt on them (in case they don't start moving at the same time).

You will find a very good summary and a few problems of free fall with solutions in the page from where this last image was obtained. Just click on the link and take a look.