p93. 4

a) $\left.\begin{array}{l} 3^{x+1} + 4^{y-2} = 5 \\ 3^{x+2} + 4^{y-1} = 19 \end{array}\right\} \longrightarrow \left\{\begin{array}{l} 3^x \cdot 3 + \frac{4^y}{4^2} = 5 \\ \longrightarrow 3^x \cdot 3^2 + \frac{4^y}{4} = 19 \end{array}\right\}$

Quita denominadores $\xrightarrow{(\cdot 4^2)}$ $3^x \cdot 3 \cdot 4^2 + 4^y = 5 \cdot 4^2 \longrightarrow 48 \cdot 3^x + 4^y = 80$

$\xrightarrow{(\cdot 4)}$ $3^x \cdot 3^2 \cdot 4 + 4^y = 19 \cdot 4 \longrightarrow 36 \cdot 3^x + 4^y = 76$ $\xrightarrow{\cdot(-1)}$

$\longrightarrow 48 \cdot 3^x + 4^y = 80$

$\xrightarrow{\cdot(-1)} -36 \cdot 3^x - 4^y = -76$

$12 \cdot 3^x = 4$

$3^x = \frac{4}{12} = \frac{1}{3} = 3^{-1}$

$\boxed{x = -1}$ $\longrightarrow$ $3^{-1+1} + 4^{y-2} = 5$

$1 + 4^{y-2} = 5$

$4^{y-2} = 4$

$y - 2 = 1$

$\boxed{y = 3}$

Sol: $x = -1$, $y = 3$

b) $\left.\begin{array}{l}\log_2 x^2 - \log_2 y = 2 \\ \log_2\left(\frac{x^3}{y}\right) = 6\end{array}\right\} \rightarrow \left.\begin{array}{l}\log_2\left(\frac{x^2}{y}\right) = 2 \\ \log_2\left(\frac{x^3}{y}\right) = 6\end{array}\right\} \rightarrow$

$\rightarrow \left.\begin{array}{l} 2^2 = \frac{x^2}{y} \\ 2^6 = \frac{x^3}{y}\end{array}\right\} \rightarrow \left.\begin{array}{l} 4y = x^2 \\ 64y = x^3\end{array}\right\} \rightarrow y = \frac{x^2}{4}$

$\downarrow$

$\rightarrow 64 \cdot \frac{x^2}{4} = x^3$

$16x^2 = x^3$

$x^3 - 16x^2 = 0$

$x^2(x-16) = 0$

$x = 0 \quad x = 16$

- ~~$x = 0 \rightarrow y = 0$~~

- $x = 16 \rightarrow y = \frac{16^2}{4} = 64$

- Comprobación (por tener logaritmos)

- ~~$x = 0$~~ $\rightarrow \log_2 0^2 - \log_2 0$ ∄ NO SOL.
  ~~$y = 0$~~

- $x = 16$, $y = 64$
  - $\log_2 16^2 - \log_2 64 = \log_2 2^8 - \log_2 2^6 = 8 - 6 = 2$ Sí
  - $\log_2\left(\frac{16^3}{64}\right) = \log_2\left(\frac{2^{12}}{2^6}\right) = \log_2 2^6 = 6$ Sí

Solución

c) $\left.\begin{array}{l} 2^{x+1} + 5^{y-2} = 5 \\ 4^x - 5^{y-1} = -1 \end{array}\right\} \begin{array}{l} \rightarrow 2^x \cdot 2 + \frac{5^y}{5^2} = 5 \\ \rightarrow 2^{2x} - \frac{5^y}{5} = -1 \end{array}$

$\xrightarrow{\cdot 25}$ $2^x \cdot 2 \cdot 25 + 5^y = 5 \cdot 25$ $\rightarrow 50 \cdot 2^x + 5^y = 125$

$\xrightarrow{\cdot 5}$ $5 \cdot 2^{2x} - 5^y = -1 \cdot 5$ $\rightarrow 5 \cdot 2^{2x} - 5^y = -5$

$$5 \cdot 2^{2x} + 50 \cdot 2^x = 120$$

$$5 \cdot 2^{2x} + 50 \cdot 2^x - 120 = 0$$

· Cambio obla. $2^x = t$

$$5 \cdot t^2 + 50t - 120 = 0$$

$:5 \hookrightarrow t^2 + 10t - 24 = 0$

$$t = \frac{-10 \pm \sqrt{100+96}}{2} = \frac{-10 \pm 14}{2} \begin{array}{l} \nearrow 2 \\ \searrow -12 \end{array}$$

· Deshago el cambio de obla:

· $t = 2 \rightarrow 2^x = 2 \rightarrow \boxed{x = 1}$

· $t = -12 \rightarrow 2^x \neq -12$ No sol.

Si $x = 1 \rightarrow 2^{1+1} + 5^{y-2} = 5$

$4 + 5^{y-2} = 5$

$5^{y-2} = 1 \rightarrow y - 2 = 0$

$\boxed{y = 2}$

Sol: $x = 1$, $y = 2$

d) $\left.\begin{array}{l} \log x^2 + 2\log y = 2 \\ x - y = 3 \end{array}\right\} \xrightarrow{\text{Prop. exponente}} \left.\begin{array}{l} 2\cdot\log x + 2\log y = 2 \\ x - y = 3 \end{array}\right\}$

$\xrightarrow{:2} \left.\begin{array}{l} \log x + \log y = 1 \\ x - y = 3 \end{array}\right\} \xrightarrow{\text{Prop. suma}} \left.\begin{array}{l} \log(x\cdot y) = 1 \\ x - y = 3 \end{array}\right\}$

$\xrightarrow{\text{Def}} \left.\begin{array}{l} 10^1 = x\cdot y \\ x - y = 3 \end{array}\right\} \rightarrow x = y+3 \rightarrow 10 = (y+3)\,y$

$y^2 + 3y - 10 = 0$

$y = \frac{-3 \pm \sqrt{9+40}}{2} = \frac{-3 \pm 7}{2} \begin{cases} 2 \\ -5 \end{cases}$

- Si $y = 2 \rightarrow x = 2+3 = 5$
- Si $y = -5 \rightarrow x = -5+3 = -2$.

Caup (por haber log es obligatorio)

- $\boxed{\begin{array}{l} x = 5 \\ y = 2 \end{array}}$ $\cdot \log 5^2 + 2\cdot\log 2 = \log 5^2 + \log 2^2 = \log 5^2\cdot 2^2 =$
  $= \log 100 = 2$ Sí.
  Solución
  $\cdot\ 5 - 2 = 3$ Sí

- ~~$x = -2$~~ $\cdot \log(-2)^2 + 2\log(-5)$
  ~~$y = -5$~~
  No sol.

p93:6

a) $\left.\begin{array}{l} x^2+y^2=5 \\ x^2-y^2=3 \end{array}\right\}$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2 \longrightarrow y^2 = 5-x^2$ $\begin{cases} x=2 \to y=\pm\sqrt{5-2^2}=\pm 1 \\ x=-2 \to y=\pm\sqrt{5-(-2)^2}=\pm 1 \end{cases}$

Sol:	$x=2, y=1$
	$x=2, y=-1$
	$x=-2, y=1$
	$x=-2, y=-1$

b) $\left.\begin{array}{l} x+y=2 \\ x^2+2y=7 \end{array}\right\}$ $\cdot(-2)$ $\left.\begin{array}{l} -2x-2y=-4 \\ x^2+2y=7 \end{array}\right\}$

$x^2-2x = 3$

$x^2-2x-3=0$

$x = \frac{2\pm\sqrt{4+12}}{2} = \frac{2\pm 4}{2} \begin{cases} 3 \\ -1 \end{cases}$

- $x=3 \to y=2-x=2-3=-1$
- $x=-1 \to y=2-(-1)=3$

Sol: $x=3, y=-1$; $x=-1, y=3$

c) $\left.\begin{array}{l} 2x+y=3 \\ x^2-y^2=0 \end{array}\right\} \longrightarrow y=3-2x$

$\downarrow$

$x^2-(3-2x)^2=0$

$x^2-(9+4x^2-12x)=0$

$x^2-9-4x^2+12x=0$

$-3x^2+12x-9=0$

$:(-3)$

$x^2-4x+3=0$

$x=\frac{4\pm\sqrt{16-12}}{2}=\frac{4\pm 2}{2}\begin{array}{l}/\,3\\ \backslash\,1\end{array}$

- $x=3 \rightarrow y=3-2\cdot 3=-3$
- $x=1 \rightarrow y=3-2\cdot 1=1$

Sol:

$x=3,\ y=-3$
$x=1,\ y=1$

d) $\left.\begin{array}{l} 3x+y=1 \\ xy=-2 \end{array}\right\} \longrightarrow y=1-3x$

$\downarrow$

$x\cdot(1-3x)=-2$

$x-3x^2=-2$

$3x^2-x-2=0$

$x=\frac{1\pm\sqrt{1+24}}{6}=\frac{1\pm 5}{6}\begin{array}{l}/\,1\\ \backslash\,-\frac{2}{3}\end{array}$

- $x=1 \rightarrow y=1-3\cdot 1=-2$
- $x=-\frac{2}{3} \rightarrow y=1-3\cdot\left(-\frac{2}{3}\right)=3$

Sol:

$x=1,\ y=-2$
$x=-\frac{2}{3},\ y=3$

e) $\left.\begin{matrix} 3x^2-5y^2=30 \\ x^2-2y^2=7 \end{matrix}\right\} \xrightarrow{\cdot(-3)} \left.\begin{matrix} 3x^2-5y^2=30 \\ -3x^2+6y^2=-21 \end{matrix}\right\}+$

$y^2=9$

$\boxed{y=\pm 3}$

- $y=3 \rightarrow x^2=7+2y^2$

$x^2=7+2\cdot 3^2=7+18=25$

$x=\pm 5$

- $y=-3 \rightarrow x^2=7+2\cdot(-3)^2$

$x=25$

$x=\pm 5.$

Sol:	$x=5,\ y=3$
	$x=5,\ y=-3$
	$x=-5,\ y=3$
	$x=-5,\ y=-3$

f) $\left.\begin{matrix} \sqrt{x}+\sqrt{y}=5 \\ \sqrt{x}-\sqrt{y}=1 \end{matrix}\right\}$

$2\sqrt{x}=6$

$\sqrt{x}=3$

$()^2$

$x=3^2$

$\boxed{x=9} \rightarrow \sqrt{y}=5-\sqrt{x}$

$\sqrt{y}=5-\sqrt{9}=5-3=2$

$()^2$

$\boxed{y=2^2=4}$

g) $\left.\begin{array}{l} \frac{1}{x^2}+\frac{1}{y^2}=17 \\ \left(\frac{1}{x}+\frac{1}{y}\right)^2=9 \end{array}\right\} \rightarrow \left.\begin{array}{l} \frac{1}{x^2}+\frac{1}{y^2}=17 \\ \frac{1}{x^2}+\frac{1}{y^2}+\frac{2}{xy}=9 \end{array}\right| \cdot(-1) \rightarrow$

$$\rightarrow \left.\begin{array}{l} \frac{1}{x^2}+\frac{1}{y^2}=17 \\ -\frac{1}{x^2}-\frac{1}{y^2}-\frac{2}{xy}=-9 \end{array}\right| \cdot(-1)$$

$$-\frac{2}{xy}=8 \xrightarrow{:2} \frac{-1}{xy}=4 \rightarrow y=\frac{-1}{4x}$$

$\downarrow$ en (I)

Sustituyo en la 1ª ec:

$$\frac{1}{x^2}+\frac{1}{\left(\frac{-1}{4x}\right)^2}=17$$

$$\frac{1}{x^2}+\frac{1}{\frac{1}{16x^2}}=17$$

$$\frac{1}{x^2}+16x^2=17$$

$$1+16x^4=17x^2$$

$$16x^4-17x^2+1=0$$

Cambio vble: $x^2=t$

$$16t^2-17t+1=0$$

$$t=\frac{17\pm\sqrt{289-64}}{32}=\frac{17\pm15}{32} \begin{array}{l} \nearrow 1 \\ \searrow \frac{2}{32}=\frac{1}{16} \end{array}$$

Deshago el cambio de vble:

$t=1 \rightarrow x=\pm 1$

$t=\frac{1}{16} \rightarrow x=\pm\frac{1}{4}$

- $x=1 \rightarrow y=\frac{-1}{4x} \rightarrow y=\frac{-1}{4}$
- $x=-1 \rightarrow y=\frac{1}{4}$
- $x=\frac{1}{4} \rightarrow y=-1$
- $x=-\frac{1}{4} \rightarrow y=1$

h) $\left.\begin{array}{l} \dfrac{2}{x+1}+\dfrac{3}{y+1}=1 \\ \dfrac{4}{x-1}-\dfrac{1}{y-3}=1 \end{array}\right\}$ (I) (II)

(I) $2(y+1)+3(x+1)=(x+1)(y+1)$

$2y+2+3x+3=xy+x+y+1$

$xy-2x-y-4=0$

II)

$4(y-3)-(x-1)=(x-1)(y-3)$

$4y-12-x+1=xy-3x-y+3$

$xy-2x-5y+14=0$

$\Downarrow$

$\left.\begin{array}{l} xy-2x-y-4=0 \\ xy-2x-5y+14=0 \end{array}\right\}$ Le cambio el signo a una de ellas y sumo, desaparece la x, tanto el término solo como el xy:

$\cdot(-1)\rightarrow$ $\left.\begin{array}{l} -xy+2x+y+4=0 \\ xy-2x-5y+14=0 \end{array}\right\}$

$-4y+18=0$

$y=\frac{18}{4}=\frac{9}{2}\rightarrow$

$x\cdot\frac{9}{2}-2x-\frac{9}{2}-4=0$

$\frac{5x}{2}=\frac{17}{2}\rightarrow \boxed{x=\frac{17}{5}}$