

Calcula:

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \dots + \frac{1}{\sqrt{2021} + \sqrt{2022}}$$

Para resolverlo, racionalizaremos un sumando genérico

$$\begin{aligned}\frac{1}{\sqrt{a} + \sqrt{a+1}} &= \frac{1 \cdot (\sqrt{a} - \sqrt{a+1})}{(\sqrt{a} + \sqrt{a+1}) \cdot (\sqrt{a} - \sqrt{a+1})} = \frac{\sqrt{a} - \sqrt{a+1}}{a - (a+1)} = \\ &= \frac{\sqrt{a} - \sqrt{a+1}}{-1} = -\sqrt{a} + \sqrt{a+1}\end{aligned}$$

Luego

$$\frac{1}{\sqrt{1} + \sqrt{2}} = -\sqrt{1} + \sqrt{2}$$

$$\frac{1}{\sqrt{2} + \sqrt{3}} = -\sqrt{2} + \sqrt{3}$$

....

$$\frac{1}{\sqrt{2020} + \sqrt{2021}} = -\sqrt{2020} + \sqrt{2021}$$

$$\frac{1}{\sqrt{2021} + \sqrt{2022}} = -\sqrt{2021} + \sqrt{2022}$$

Si sustituimos tenemos

$$-\sqrt{1} + \sqrt{2} - \sqrt{2} + \sqrt{3} - \dots - \sqrt{2020} + \sqrt{2021} - \sqrt{2021} + \sqrt{2022}$$

Simplificando queda

$$-\sqrt{1} + \sqrt{2022} = -1 + \sqrt{2022}$$

Por tanto

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \dots + \frac{1}{\sqrt{2021} + \sqrt{2022}} = -1 + \sqrt{2022}$$

