1

Los n^{os} pares $\Rightarrow (2n)^m = 2^m \cdot n^m \rightarrow$ son pares

veamos los impares:

son impares
- los que acaban en 1 → sus potencias acaban en 1
- los que acaban en 3 → sus potencias acaban en 3, 9, 7, 1
- los que acaban en 5 → sus potencias acaban en 5
- los que acaban en 7 → sus potencias acaban en 7, 9, 3, 1
- los que acaban en 9 → sus potencias acaban en 9, 1

- $3^1 = 3,\ 3^2 = 9,\ 3^3 = 27,\ 3^4 = 81,\ 3^5 = 243 \ldots$
- $5^1 = 5,\ 5^2 = 25;\ 5^3 = 125,\ 5^4 = 625 \ldots$
- $7^1 = 7,\ 7^2 = 49,\ 7^3 = 343,\ 7^4 = 2401,\ 7^5 = 16807$
- $9^1 = 9,\ 9^2 = 81,\ 9^3 = 729, \ldots$

Par + impar = Impar $\Rightarrow$ No hay ninguna suma que de par

↳ a)

2

24 pág —— 1'5 min

x —— 5 min

$x = \frac{25 \cdot 4}{1'5} = 80 \rightarrow$ c)

3

x?

$\left(\frac{x-1+2}{3}\right) \cdot 4 = 56 \rightarrow x = \frac{56 \cdot 3}{4} + 1 - 2 = 41 \rightarrow$ d)

4

total 20 partidas

gana $9 + 5 = 14$

$\Rightarrow \frac{14}{20} \cdot 100 = 70\% \Rightarrow$ c)

(5)

	c	d
a	12	x
b	20	30

$a \cdot c = 12 \Rightarrow 3 \cdot 4 = 12$ (a = 3)

$b \cdot c = 20 \quad \boxed{5} \cdot 4 = 20$

$b \cdot d = 30 \Rightarrow \boxed{5} \cdot d = 30$

$d = \frac{30}{5} = 6$

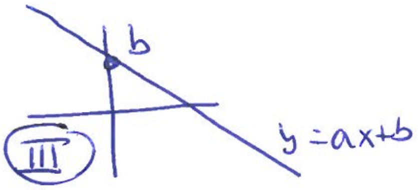
Área $= a \cdot d = 3 \cdot 6 = 18$ cm² $\Rightarrow$ b)

(6)

$y = -2017x + 2018$

$y = ax + b$ $\quad a < 0$ decrece

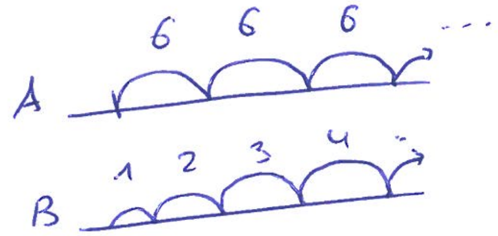
$(0, b)$ ordenada origen



III

↳ c)

(7)



$\Rightarrow 6 \cdot n$

$\Rightarrow 1 + 2 + 3 + 4 + \dots + n = \frac{(1+n) \cdot n}{2}$

↑ Suma n términos P.A. $d = 1$, $a_1 = 1$

recorren misma distancia

$6n = \frac{(1+n) \cdot n}{2} \rightarrow 12n = n + n^2$

$0 = n^2 - 11n$

$0 = n \cdot (n - 11)$ → $n = 0$ # ; $\boxed{n = 11} \Rightarrow$ c)

(8)

B → $42 = 2 \cdot 3 \cdot 7$

A → $66 = 2 \cdot 3 \cdot 11$

R → $78 = 2 \cdot 3 \cdot 13$

MCD (42, 66, 78) = 6

Semillas habrá $66 : 6 = \boxed{11} \rightarrow$ d)

(9)

$a + b + c + d + e = 17$

$a \cdot b \cdot c \cdot d \cdot e =$ máx

$1 + 2 + 3 + 4 + 7 = 17 \rightarrow 1 \cdot 2 \cdot 3 \cdot 4 \cdot 7 = 168$

$1 + 2 + 3 + 5 + 6 = 17 \rightarrow 1 \cdot 2 \cdot 3 \cdot 5 \cdot 6 = 180 \rightarrow$ e)

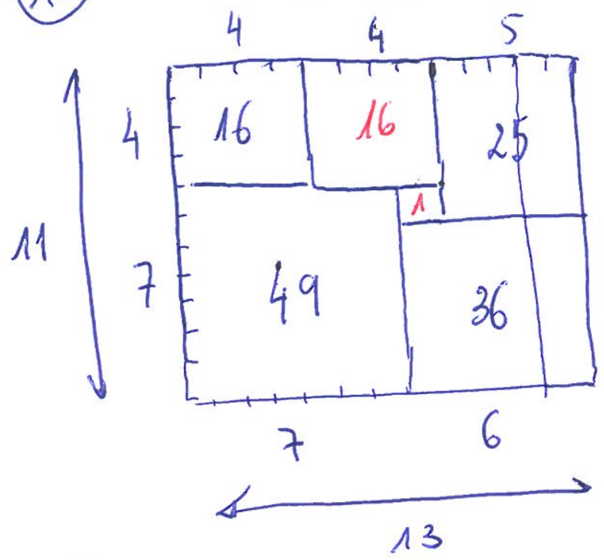
no son posibles las sumas con:

$9 + 8 = 17$ → falta un sumando

$8 + 7 = 15$ → se repetirían.

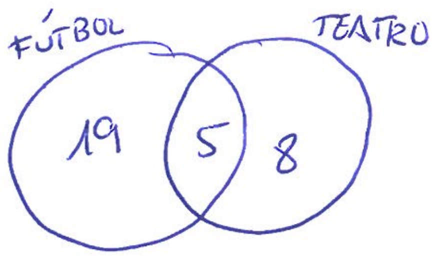
$7 + 6 = 13$ → se repetirían.

(10) CANGUR 2018 1 BAT (2)



$$A = 13 \cdot 11 = 143 \, cm^2 \Rightarrow \underline{\underline{b)}}$$

(11)

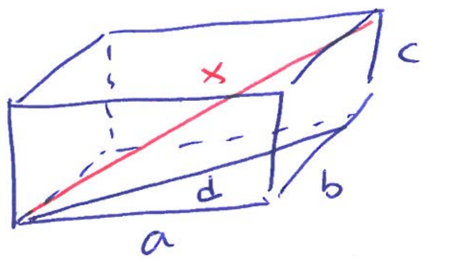


TOTAL: $19 + 5 + 8 = 19 + 13 = 32 \rightarrow \underline{\underline{d)}}$

(12) $$\left(\left(\sqrt{2}\right)^{\sqrt{2}}\right)^{\sqrt{2}} \underset{\uparrow}{=} \left(\sqrt{2}\right)^{\sqrt{2}\cdot\sqrt{2}} = \left(\sqrt{2}\right)^{2} = 2 \Rightarrow a)$$

$$(a^m)^n = a^{m \cdot n}$$

(13)



$d^2 = a^2 + b^2$

$x^2 = d^2 + c^2$

$\downarrow$

$x^2 = a^2 + b^2 + c^2$

Cabrá si $x^2 \leq a^2 + b^2 + c^2$

a) $2^2 + 2^2 + 10^2 = 4 + 4 + 100 = 108 \geq 100$

b) $2^2 + 5^2 + 9^2 = 4 + 25 + 81 = 110 \geq 100$

c) $2^2 + 6^2 + 8^2 = 4 + 36 + 64 = 104 \geq 100$

d) $3^2 + 5^2 + 8^2 = 9 + 25 + 64 = 98 < 100 \rightarrow \underline{\underline{d)}}$

e) $4^2 + 6^2 + 7^2 = 16 + 36 + 49 = 101 \geq 10$

(14) $ab0 = \boxed{a^3+b^3 = 100a + 10b}$ *

a) $100a + b = a^3+b^3$

b) $(100)\cdot(a-1) + b = 100a - 100 + b = (a-1)^3 + b^3$

c) $100a + (b-1)\cdot 10 = 100a + 10b - 10 = a^3 + (b-1)^3$

d) $100a + 10b + 1 = a^3+b^3 + 1 = \underbrace{100a+10b}+1$ ✓ $\Rightarrow$ d)

e) $100a + 10b + b = a^3 + 1^3 + b^3$

(15) $w+x=3$

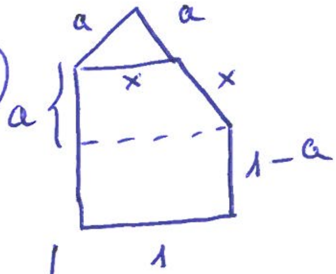
$\frac{1}{w} + \frac{1}{x} = 1 \Rightarrow \frac{x+w}{w\cdot x} = 1 \Rightarrow \frac{3}{wx} = 1 \rightarrow \boxed{wx=3}$ (b)

$(w+x)^2 = w^2+x^2+2wx \Rightarrow 3^2 = w^2+x^2+2\cdot 3 \Rightarrow w^2+x^2 = 9-6$

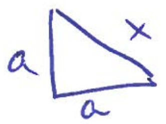
$\boxed{w^2+x^2=3}$ (a)

calculo $\frac{w}{x} + \frac{x}{w} = \frac{w^2+x^2 \text{ (a)}}{x\cdot w \text{ (b)}} = \frac{3}{3} = \boxed{1}$ $\rightarrow$ b)

(16)



El pentágono y el cuadrado tienen la misma área $\boxed{1\ dm^2}$



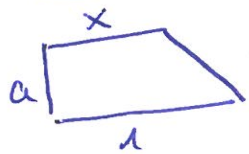
$\rightarrow$ pitágoras $x^2 = a^2+a^2$

$x^2 = 2a^2$

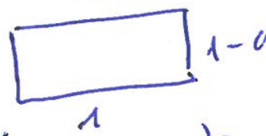
$\boxed{x = \sqrt{2}\cdot a}$ *

Calculamos área pentágono por trozos:

$\Rightarrow A = \frac{a\cdot a}{2} = \frac{a^2}{2}$



$\Rightarrow A = \frac{(x+1)\cdot a}{2} \overset{*}{=} \frac{(\sqrt{2}\cdot a+1)\cdot a}{2} = \frac{\sqrt{2}a^2+a}{2}$



$\Rightarrow A = 1\cdot(1-a) = 1-a$

La suma da $1\ dm^2 \Rightarrow \frac{a^2}{2} + \frac{\sqrt{2}a^2+a}{2} + \cancel{1} - a = \cancel{1} \Rightarrow \frac{a^2}{2} + \frac{\sqrt{2}a^2+a}{2} - \frac{2a}{2} = 0 \Rightarrow$

$\Rightarrow a^2 + \sqrt{2}a^2 - a = 0 \quad a\cdot[a+\sqrt{2}a-1] = 0$ $\nearrow a = 0$ ✗ $\searrow a+\sqrt{2}a-1=0$ $\rightarrow \boxed{a = \frac{1}{1+\sqrt{2}}}$

racionalizamos $\frac{1}{1+\sqrt{2}} = \frac{1}{1+\sqrt{2}}\cdot\frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-\sqrt{2}}{1^2-(\sqrt{2})^2} = \frac{1-\sqrt{2}}{-1} = \boxed{\sqrt{2}-1}$

$\rightarrow$ a)

17 nº diagonales de un polígono $\frac{n\cdot(n-3)}{2}$

polígono de n lados

$$\frac{n(n-3)}{2}+n=28$$

$$\frac{n^2-3n+2n}{2}=28 \rightarrow n^2-3n+2n=56$$
$$n^2-n-56=0 \Rightarrow$$

$$\Rightarrow n=\frac{1\pm\sqrt{(-1)^2-4.1.(-56)}}{2.1}=\frac{1\pm 15}{2} < \begin{matrix}\boxed{8} \longrightarrow \text{b)} \\ -7 \text{ ∄}\end{matrix}$$

18 $A_{círculo}=36\pi$

$\pi R^2=36\pi \rightarrow R^2=36 \rightarrow R=6$ cm

$L=2\pi R=2\pi.6=12\pi$

Figura: $\frac{3}{4}L+2R=\frac{3}{4}.12\pi+2.6=9\pi+12 \rightarrow$ b)

19 $\{1 --- 17\}$

d = 1 ⇒ $\{1,2,3,4,5\}$ ⋮ $\{13,14,15,16,17\}$ } hay 13

d = 2 ⇒ $\{1,3,5,7,9\}$ ⋮ $\{9,11,13,15,17\}$ } hay 9

d = 3 ⇒ $\{1,4,7,10,13\}$ ⋮ $\{5,8,11,14,17\}$ } hay 5

d = 4 → $\{1,5,9,13,17\}$ → hay 1

en total $13+9+5+1=28$ → e)

20 $x>2016.2018$ → $x>4068288$
$x<2017.2019$ → $x<4072323$ } resto $=4035-1=4034 \Rightarrow$ d)

¿por qué resto 1? fíjate

$3<x<5 \longrightarrow 5-3=2 \rightarrow$ pero sólo está $\{4\}$ 1

$3<x<6 \longrightarrow 6-3=3 \rightarrow$ pero sólo están $\{4,5\}$ 2

es decir hay 1 menos.

(21) $\frac{1}{5}$ menos $\rightarrow$ 0'2 menos $\rightarrow$ la vuelta va a una velocidad del 80% de la ida.

$\boxed{v_B = 0'8 \cdot v_A}$

CASA $\xrightarrow[t]{\vec{v}_A}$ FUENTE

$\xleftarrow[(t+1)]{\vec{v}_B}$

$$e = v \cdot t$$

Ida — vuelta

$$v_A \cdot t = v_B \cdot (t+1)$$
$$\cancel{v_A} \cdot t = 0'8 \cancel{v_A} \cdot (t+1)$$
$$t = 0'8t + 0'8$$
$$t - 0'8t = 0'8$$
$$0'2t = 0'8 \rightarrow \boxed{t = 4\,h} \text{ la ida.}$$

total

$$\underbrace{t}_{\text{ida}} + \underbrace{1}_{\text{descanso}} + \underbrace{t+1}_{\text{vuelta}} = 4+1+4+1 = 10\,h \rightarrow \underline{\underline{e)}}$$

(22)

65535	3
21845	5
4369	17
257	257
1	

(17)

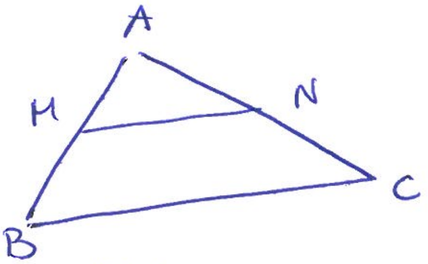
$3 \cdot 5 = 15$

$3 \cdot 17 = 51$

$5 \cdot 17 = 85$

4 divisores de 2 cifras $\rightarrow$ $\underline{\underline{d)}}$

(23)



$AB = 16$

$BC = 12$

$AC = 8$

$P_{\triangle ABC} = 16+12+8 = 36$

$P_{\triangle AMN} = 27$

cte de proporcionalidad

$$K = \frac{P_{\triangle ABC}}{P_{\triangle MN}} = \frac{36}{27} = \boxed{\frac{4}{3}}$$

$$\frac{AB}{AM} = \frac{4}{3} \rightarrow AM = \frac{3 \cdot AB}{4} = \frac{3 \cdot 16}{4} = 12\,cm \rightarrow MB = 16-12 = 4\,cm$$

$$\frac{BC}{MN} = \frac{4}{3} \rightarrow MN = \frac{3 \cdot BC}{4} = \frac{3 \cdot 12}{4} = 9\,cm$$

$$\frac{AC}{AN} = \frac{4}{3} \rightarrow AN = \frac{3 \cdot AC}{4} = \frac{3 \cdot 8}{4} = 6\,cm \rightarrow NC = AC - AN = 8-6 = 2\,cm$$

$$P_{MNBC} = MN + MB + NC + BC = 9 + 4 + 2 + 12 = \boxed{27} \Rightarrow \underline{\underline{a)}}$$

24) $21 \begin{cases} 20 \times 652 \\ 1 \times 692 \end{cases} \oplus \; 13692 \Rightarrow$ media $\frac{13692}{21} = \boxed{652} \Rightarrow$ c)

25) 1 recta pasa por 2 puntos

$C_{10,2} = \binom{10}{2} = \frac{10!}{8!\,2!} = 45$ rectas

tengo que eliminar las rectas que coinciden (alineados)

$C_{4,2} = \binom{4}{2} = \frac{4!}{2!\cdot 2!} = 6 \longrightarrow$ pero de todas me quedo con 1 la que pasa por los 4 puntos

TOTAL: $45 - 6 + 1 = 40 \Rightarrow$ b)

26) $x^2 + x - 7 = 0 \rightarrow x = \frac{-1 \pm \sqrt{1+28}}{2} = \frac{-1 \pm \sqrt{29}}{2}$

$a_1 = \frac{-1+\sqrt{29}}{2} \longrightarrow a_1^2 = \frac{1+29-2\sqrt{29}}{4} = \frac{30-2\sqrt{29}}{4} = \frac{15-\sqrt{29}}{2}$

$b_1 = \frac{-1-\sqrt{29}}{2} \longrightarrow b_1^2 = \frac{1+29+2\sqrt{29}}{4} = \frac{15+\sqrt{29}}{2}$

(también podría ser $a_2 = \frac{-1-\sqrt{29}}{2}$, $b_2 = \frac{-1+\sqrt{29}}{2}$)

comprobemos cuál es.

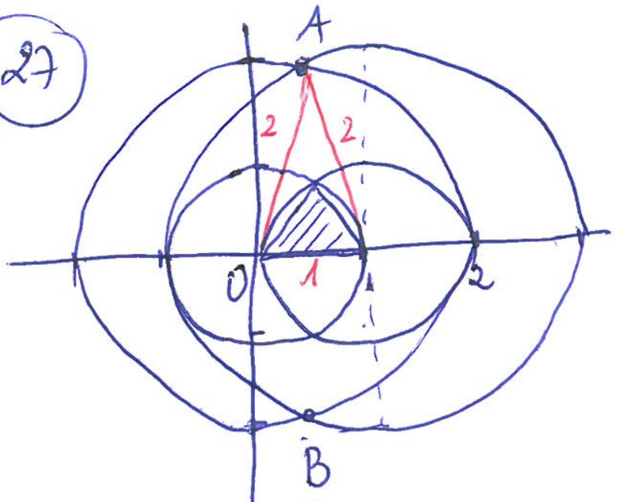
1ª $3a_1^2 + 4b_1^2 + 2a_1 + 3b_1 + 1 = 3\left(\frac{15-\sqrt{29}}{2}\right) + 4\left(\frac{15+\sqrt{29}}{2}\right) + 2\cdot\left(\frac{-1+\sqrt{29}}{2}\right) + 3\left(\frac{-1-\sqrt{29}}{2}\right) + 1 =$

$= \frac{30 - 3\sqrt{29} + 60 + 4\sqrt{29} - 2 + 2\sqrt{29} - 3 - 3\sqrt{29} + 2}{2} = \boxed{\frac{87}{2}}$ ✗

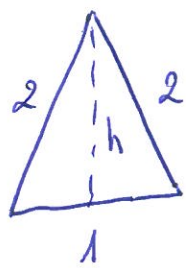
2ª $3a_2^2 + 4b_2^2 + 2a_2 + 3b_2 + 1 = 3\left(\frac{15+\sqrt{29}}{2}\right) + 4\left(\frac{15-\sqrt{29}}{2}\right) + 2\cdot\left(\frac{-1-\sqrt{29}}{2}\right) + 3\cdot\left(\frac{-1+\sqrt{29}}{2}\right) + 1 =$

$= \frac{45 + \cancel{3\sqrt{29}} + 60 - \cancel{4\sqrt{29}} - 2 - \cancel{2\sqrt{29}} - 3 + \cancel{3\sqrt{29}} + 2}{2} = \frac{102}{2} = \boxed{51}$ ✓ $\Rightarrow$ b)

27



$d(A,B) = \boxed{2h}$



$P = 2+2+1 = 5 \to \frac{P}{2} = \frac{5}{2} = p$

$A = \frac{b \cdot h}{2} = \boxed{\frac{1 \cdot h}{2}}$

Fórmula de Herón

$A = \sqrt{p(p-a)(p-b)(p-c)} =$

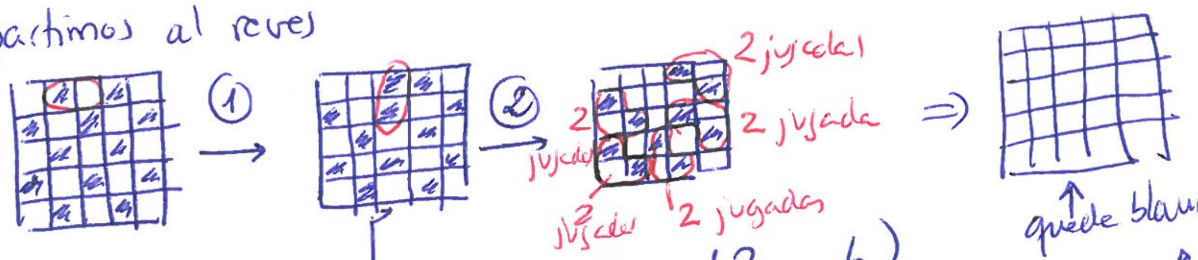
$= \sqrt{\frac{5}{2} \cdot \left(\frac{5}{2}-1\right)\left(\frac{5}{2}-2\right)\left(\frac{5}{2}-2\right)} =$

$= \sqrt{\frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}} = \boxed{\frac{\sqrt{15}}{4}}$

Igualamos $\frac{1 \cdot h}{2} = \frac{\sqrt{15}}{4} \to \boxed{h = \frac{\sqrt{15}}{2}}$

$d(A,B) = 2 \cdot \frac{\sqrt{15}}{2} = \boxed{\sqrt{15}} \Rightarrow$ c)

28 partimos al revés



para eliminar 2 casillas negras necesito 2 jugadas

$2 + 10 = \underline{\underline{12}}$ $\underline{\underline{b)}}$

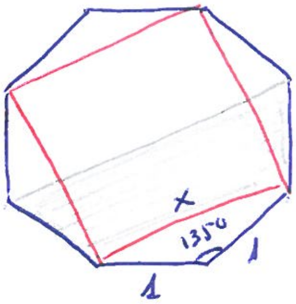
queda blanco

en total 12 jugadas para eliminar las 12 casillas negras

29

$n^n = m$

$n^{(n^{n+1})} = n^{n^n \cdot n} = n^{m \cdot n} = (n^n)^m = m^m \to \underline{\underline{d)}}$



Supongamos que el lado mide 1cm

Suma ángulos interiores

$$S_n = 180(n-2)$$
$$S_8 = 180(8-2) = 180 \cdot 6 = 1080$$

Como es regular cada ángulo mide

$$\frac{S_n}{n} \to \frac{S_8}{8} = \frac{1080}{8} = 135^\circ$$

Como piden la razón entre ambas áreas no importa la longitud con la que trabajemos por ello consideramos que el lado vale 1 y no se pierde la generalidad del ejercicio.

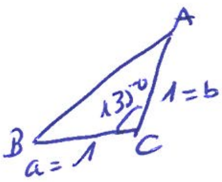
Calculamos x (base del cuadrado)

Tª del coseno

$$x^2 = 1^2 + 1^2 - 2 \cdot 1 \cdot 1 \cdot \cos 135 = 1 + 1 - \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right) = 2 + \sqrt{2}$$

Área del cuadrado $x^2 = \boxed{2+\sqrt{2}\ cm^2}$

Ahora calculamos el área del triángulo



$$A = \frac{a \cdot b \cdot \operatorname{sen} C}{2} = \frac{1 \cdot 1 \cdot \operatorname{sen} 135}{2} = \frac{1 \cdot 1 \cdot \sqrt{2}/2}{2} = \boxed{\frac{\sqrt{2}}{4}\ cm^2}$$

A continuación calculamos el área del octógono

$$A_{octógono} = A_{cuadrado} + 4\, A_{triángulo} = 2+\sqrt{2} + \cancel{4} \cdot \frac{\sqrt{2}}{\cancel{4}} = \boxed{2+2\sqrt{2}\ cm^2}$$

Calculemos Área trapecio.

$$A_{trapecio} = \frac{A_{cuadrado}}{2} + A_{triángulo} = \frac{2+\sqrt{2}}{2} + \frac{\sqrt{2}}{4} = \frac{4+2\sqrt{2}+\sqrt{2}}{4} =$$

$$= \boxed{\frac{4+3\sqrt{2}}{4}\ cm^2}$$

Por último calculemos la razón:

$$* \quad \frac{A_{trapecio}}{A_{octógono}} = \frac{\frac{4+3\sqrt{2}}{4}}{2+2\sqrt{2}} = \frac{4+3\sqrt{2}}{4 \cdot (2+2\sqrt{2})} = \frac{4+3\sqrt{2}}{4(2+2\sqrt{2})} \cdot \frac{(2-2\sqrt{2})}{(2-2\sqrt{2})} =$$

racionalizamos

$$= \frac{(4+3\sqrt{2}) \cdot (2-2\sqrt{2})}{4 \cdot (2^2 - (2\sqrt{2})^2} = \frac{8 - 8\sqrt{2} + 6\sqrt{2} - 6 \cdot (\sqrt{2})^2}{4 \cdot (4-8)} = \frac{8 - 2\sqrt{2} - 12}{4 \cdot (-4)} =$$

$$= \frac{-4-2\sqrt{2}}{-16} = \frac{4+2\sqrt{2}}{16} = \boxed{\frac{2+\sqrt{2}}{8}} \Rightarrow \underline{\underline{C)}}$$