

$$\underline{p102:21}$$

$$a) 4^{x+1} + 2^{x+3} = 320$$

$$(4=2^2)$$

$$(2^2)^{x+1} + 2^{x+3} = 320$$

$$2^{2(x+1)} + 2^{x+3} = 320$$

$$2^{2x+2} + 2^{x+3} = 320$$

$$2^{2x} \cdot 2^2 + 2^x \cdot 2^3 = 320$$

$$4 \cdot 2^{2x} + 8 \cdot 2^x - 320 = 0$$

$$\div 4 \left(2^{2x} + 2 \cdot 2^x - 80 = 0 \right)$$

• Cambio de variable: $2^x = t$

$$t^2 + 2t - 80 = 0$$

$$t = \frac{-2 \pm \sqrt{4 + 320}}{2} = \frac{-2 \pm 18}{2} \begin{matrix} / 8 \\ \backslash -10 \end{matrix}$$

• Deshago el cambio de variable:

• $t=8 \rightarrow 2^x = 8 = 2^3 \rightarrow \boxed{x=3}$

• $t=-10 \rightarrow 2^x = -10$ No tiene solución.

$$b) 4^x - 8 = 2^{x+1}$$

$$2^{2x} - 8 = 2^{x+1}$$

$$2^{2x} - 2^{x+1} - 8 = 0$$

$$2^{2x} - 2^x \cdot 2 - 8 = 0$$

$$2^{2x} - 2 \cdot 2^x - 8 = 0$$

Cambio de variable: $2^x = t$

$$t^2 - 2t - 8 = 0$$

$$t = \frac{2 \pm \sqrt{4 + 32}}{2} = \frac{2 \pm 6}{2} \begin{matrix} / 4 \\ \backslash -2 \end{matrix}$$

Deshago el cambio de variable:

$$\bullet t = 4 \rightarrow 2^x = 4 = 2^2 \rightarrow \boxed{x = 2}$$

$$\bullet t = -2 \rightarrow 2^x = -2 \text{ No tiene sol.}$$

$$c) 2^{3-x} = 5 - 2^{x-1}$$

$$2^3 \cdot 2^{-x} = 5 - 2^x \cdot 2^{-1}$$

$$\frac{2^3}{2^x} = 5 - \frac{2^x}{2}$$

$$\frac{2 \cdot 2^3}{2 \cdot 2^x} = \frac{5 \cdot 2 \cdot 2^x}{2 \cdot 2^x} - \frac{2^x \cdot 2^x}{2 \cdot 2^x}$$

$$2^4 = 10 \cdot 2^x - 2^{2x}$$

$$2^{2x} - 10 \cdot 2^x + 16 = 0$$

Cambio de variable: $2^x = t$

$$t^2 - 10t + 16 = 0$$

$$t = \frac{10 \pm \sqrt{100 - 64}}{2} = \frac{10 \pm 6}{2} \begin{matrix} / 8 \\ \backslash 2 \end{matrix}$$

Deshago el cambio de variable:

$$\bullet t = 8 \rightarrow 2^x = 8 = 2^3 \rightarrow \boxed{x = 3}$$

$$\bullet t = 2 \rightarrow 2^x = 2 \rightarrow \boxed{x = 1}$$

$$d) 3 \cdot 4^x + 9 \cdot 2^x - 30 = 0$$

$$3 \cdot 2^{2x} + 9 \cdot 2^x - 30 = 0$$

$$2^{2x} + 3 \cdot 2^x - 10 = 0$$

Cambio de vble: $2^x = t$

$$t^2 + 3t - 10 = 0$$

$$t = \frac{-3 \pm \sqrt{9+40}}{2} = \frac{-3 \pm 7}{2} \begin{array}{l} / 2 \\ \backslash -5 \end{array}$$

Después el cambio de vble:

$$\bullet t = 2 \rightarrow 2^x = 2 \rightarrow \boxed{x = 1}$$

$$\bullet t = -5 \rightarrow 2^x = -5 \text{ No tiene solución.}$$

$$e) \quad 9^x = 27 + 26 \cdot 3^x$$

$$\left(\begin{array}{l} 9 = 3^2 \\ \downarrow \end{array} \right.$$

$$3^{2x} = 27 + 26 \cdot 3^x$$

$$3^{2x} - 26 \cdot 3^x - 27 = 0$$

Cambio de variable: $3^x = t$

$$t^2 - 26t - 27 = 0$$

$$t = \frac{26 \pm \sqrt{676 + 108}}{2} = \frac{26 \pm 28}{2} \begin{array}{l} / \quad 27 \\ \backslash \quad -1 \end{array}$$

Deshago el cambio de variable:

$$\bullet t = 27 \rightarrow 3^x = 27 = 3^3 \rightarrow \boxed{x = 3}$$

$$\bullet t = -1 \rightarrow 3^x = -1 \text{ No tiene solución.}$$

$$f) 5^x - 5^{x-2} = 120\sqrt{5}$$

$$5^x - 5^x \cdot 5^{-2} = 120\sqrt{5}$$

$$5^x - \frac{5^x}{5^2} = 120\sqrt{5}$$

$$5^x - \frac{5^x}{25} = 120\sqrt{5}$$

$$\frac{25 \cdot 5^x}{25} - \frac{5^x}{25} = \frac{25 \cdot 120\sqrt{5}}{25}$$

$$25 \cdot 5^x - 5^x = 25 \cdot 120\sqrt{5}$$

$$24 \cdot 5^x = 25 \cdot 120\sqrt{5}$$

$$5^x = \frac{25 \cdot 120\sqrt{5}}{24}$$

$$5^x = 25 \cdot 5 \cdot \sqrt{5}$$

$$5^x = 5^2 \cdot 5 \cdot 5^{1/2}$$

$$5^x = 5^{2+1+\frac{1}{2}}$$

$$x = 2 + 1 + \frac{1}{2} = 3 + \frac{1}{2} = \frac{6}{2} + \frac{1}{2}$$

$$\boxed{x = \frac{7}{2}}$$

Comprobación:
(práctica de operación con radicales)

$$5^{\frac{7}{2}} - 5^{\frac{7}{2}-2} = 5^{\frac{7}{2}} - 5^{\frac{3}{2}}$$

$$= \sqrt{5^7} - \sqrt{5^3} =$$

$$= \sqrt{5^6 \cdot 5} - \sqrt{5^2 \cdot 5} =$$

$$= 5^3 \sqrt{5} - 5\sqrt{5} =$$

$$= 125\sqrt{5} - 5\sqrt{5} = 120\sqrt{5}$$